Polarization of Light, Entanglement and Bell's Inequality

AJOY GHATAK
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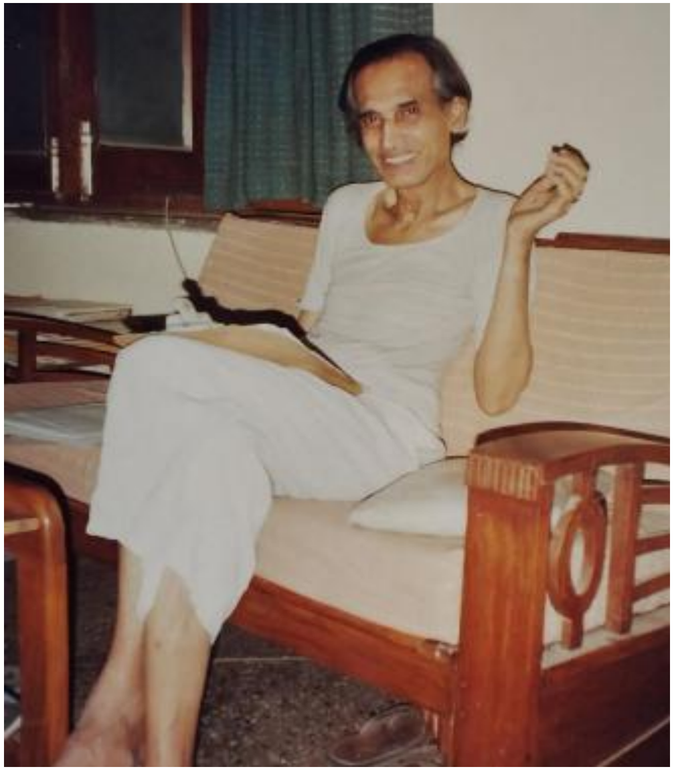
Professor Asoke Nath Mitra Memorial Meeting
April 14-15, 2025

My sincere thanks to Dr Aalok Misra and Dr Gargi Mitra for asking me to deliver a talk to this distinguished audience

My deep respect to
Professor Asoke Nath Mitra:

an outstanding physicist and a remarkably simple human being



The 2022 Nobel Prize in Physics was awarded to

Alain Aspect **John Clauser** **&** **Anton Zeilinger**

for experiments with entangled photons,

establishing the violation of Bell inequalities and

pioneering quantum information science

I will give an elementary talk on

experiments with entangled photons,

establishing the violation of Bell inequalities

National Quantum Mission (NQM)

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Human Capacity Building Programmes

National Quantum Mission (NQM)

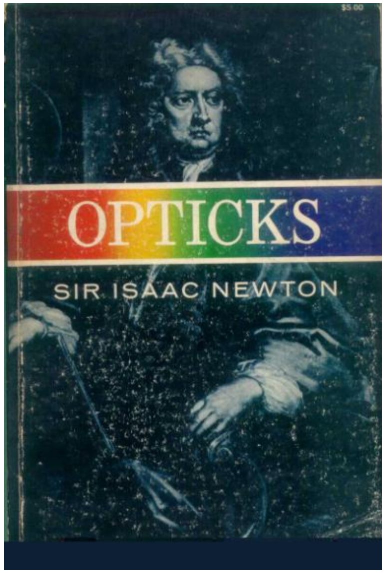
Understanding of <u>experiments with entangled photons, establishing the violation of Bell inequalities</u> .. are topics of great importance

The question

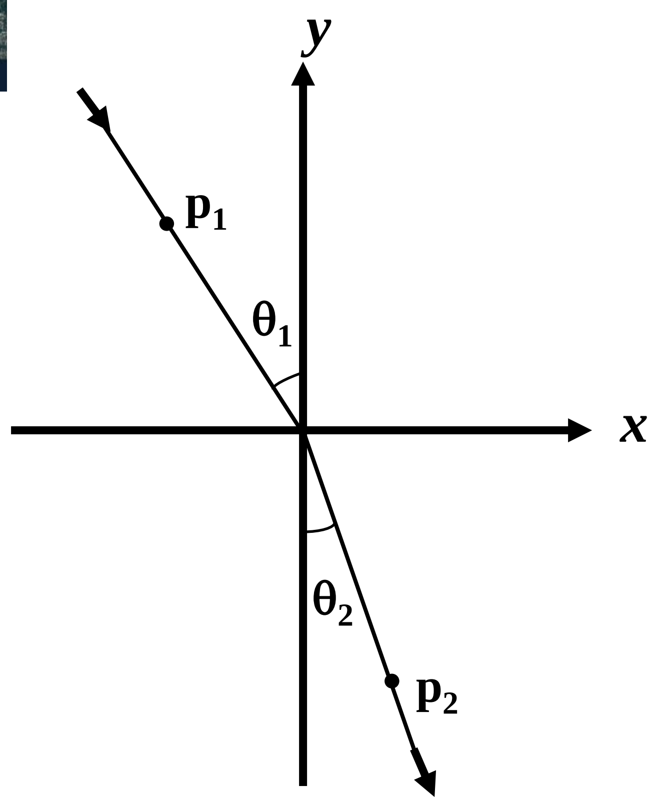
What is Light ?

has intrigued mankind

ever since he could see



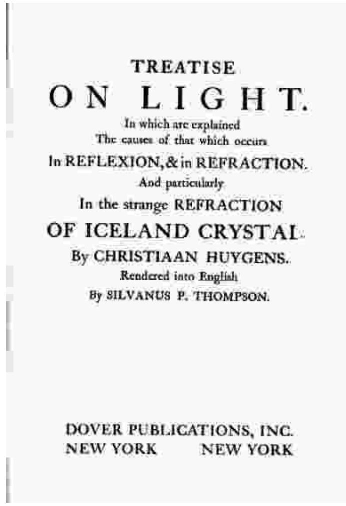
Isaac Newton in his book on *OPTICKS* (1687) had put forward the corpuscular model of light



Refraction of a corpuscle

$$\frac{\sin\,\theta_1}{\sin\,\theta_2} = \text{a constant}$$

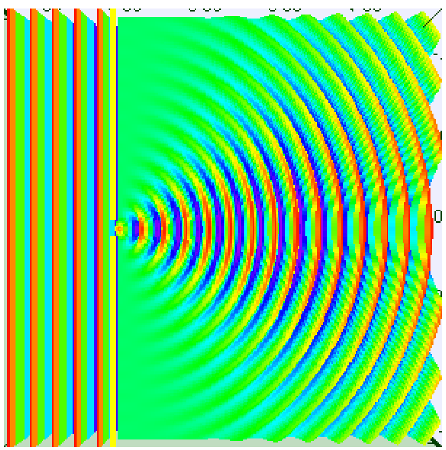
Snell's Law



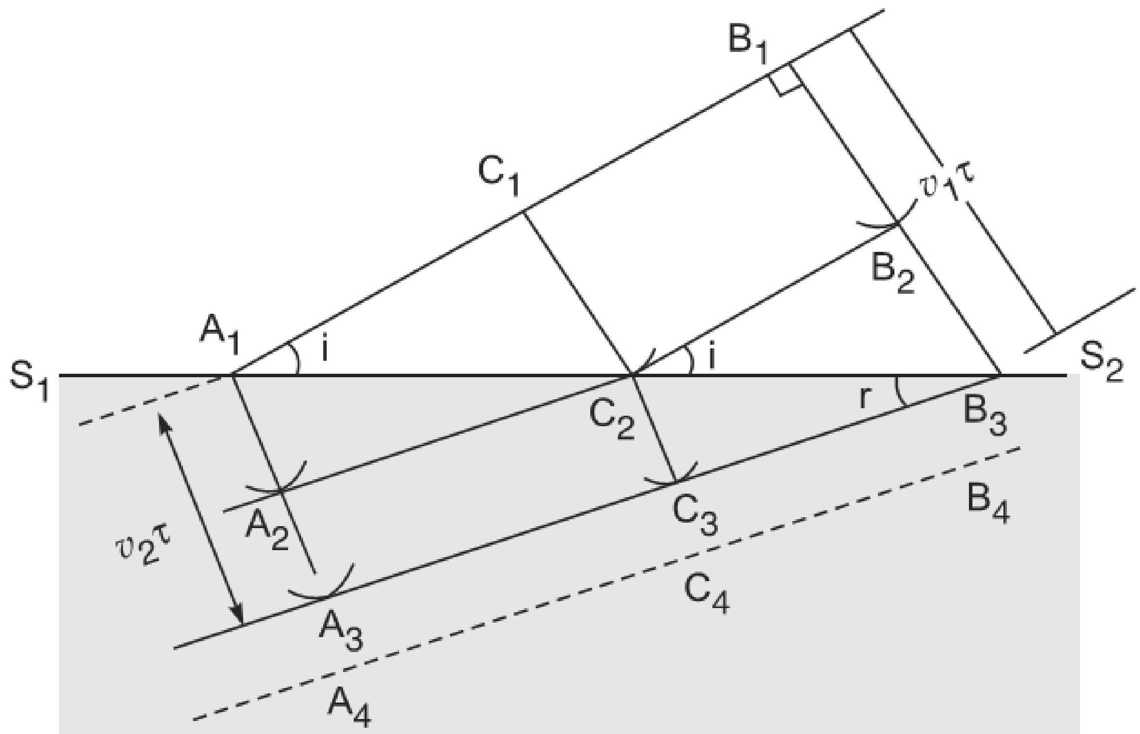
TREATISE
ON LIGHT.
In which are explained
The causes of that which occurs
In REFLEXION, & in REFRACTION.
And particularly
In the strange REFRACTION
OF ICELAND CRYSTAL.
By CHRISTIAAN HUYGENS.
Rendered into English
By SILVANUS P. THOMPSON.

DOVER PUBLICATIONS, INC.
NEW YORK NEW YORK

Around 1650, Christiaan Huygens put forward the wave theory of light.



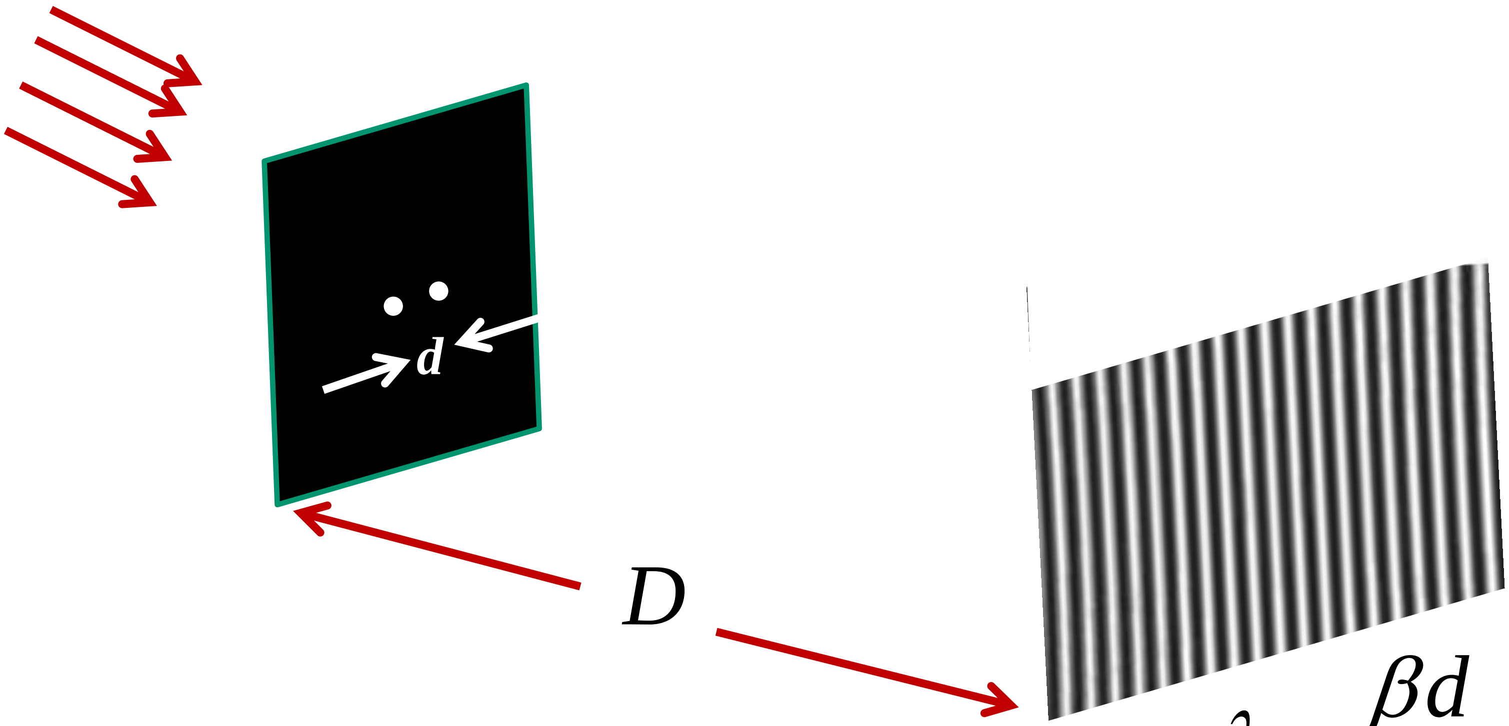
Animation by Mr Wee Loo Kang Lawrence and Mr Fu-kwun Hwang, author of EJSS 5.0 Francisco Esquembre; used with their kind permission.



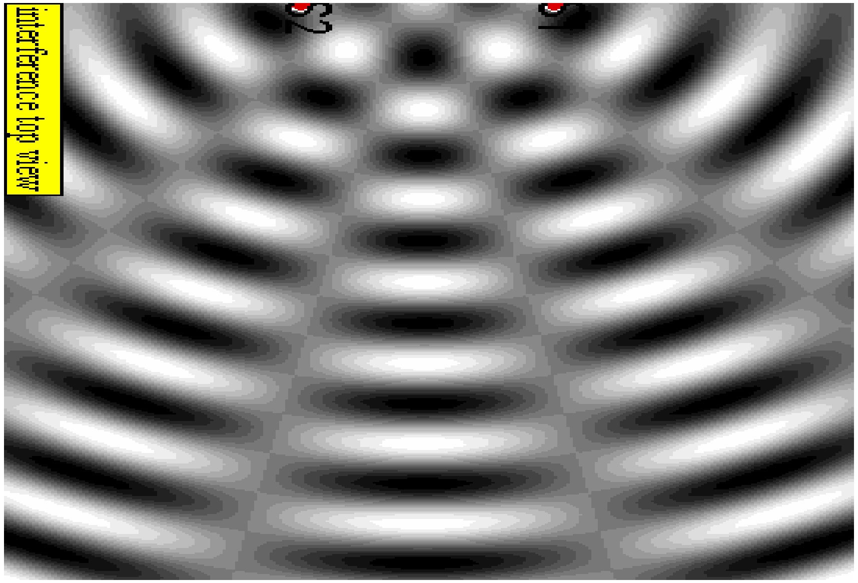
Thomas Young's Experiment (1801)

Light + Light produce darkness

Thomas Young's Experiment (1801)

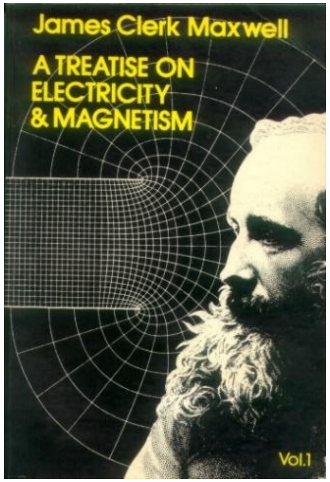


$$\lambda = \frac{\beta d}{D}$$



The interference experiment proved that light was a wave but how could it propagate through vacuum??

Lookang http://en.wikipedia.org/wiki/Ripple_tank



In free space

$$\text{curl}\ \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$ **Displacement Current**

$$\text{curl}\ \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$ **Faraday's Law**

$$\text{div}\ \mathbf{E} = 0$$

$$\text{div}\ \mathbf{H} = 0$$

$$\varepsilon_0 = 8.8542... \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \ \text{Ns}^2\ \text{C}^{-2}$$

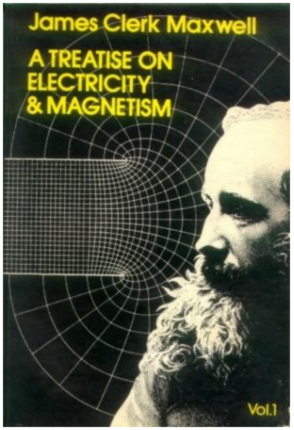
In free space

$$\left.\begin{aligned}\nabla^2\mathbf{E} &= \varepsilon_0\mu_0 \frac{\partial^2\mathbf{E}}{\partial t^2} \\ \nabla^2\mathbf{H} &= \varepsilon_0\mu_0 \frac{\partial^2\mathbf{H}}{\partial t^2}\end{aligned}\right\} \nabla^2\Psi = \frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2}$$

$$\Rightarrow v = c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \approx 311 \quad \text{million meters/s}$$

$$\Rightarrow\ v = c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 311 \quad \text{million meters/s}$$

The above value was very close to the value of the speed of light measured by Fizaeu (in 1849) which was about 318 million meters/s.

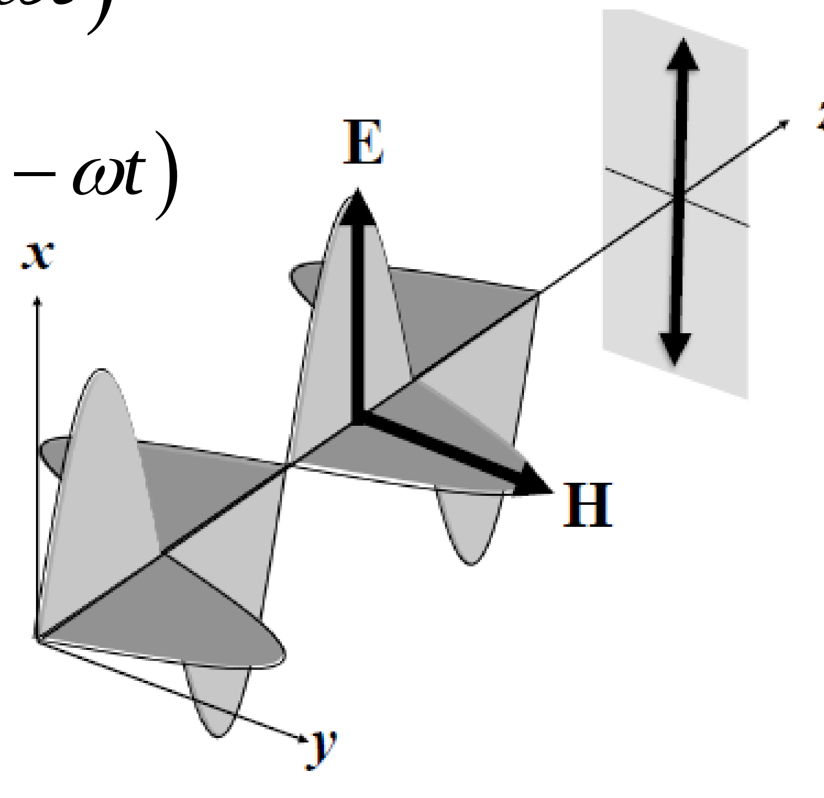


Around 1864 Maxwell predicted the existence of electromagnetic waves and said

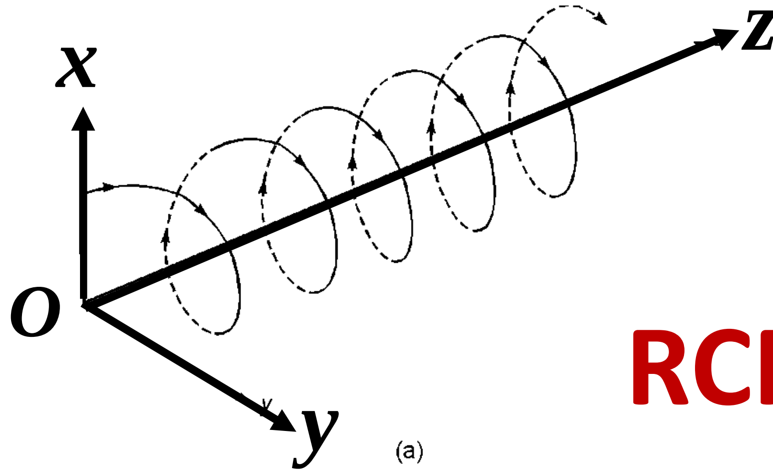
light itself is an electromagnetic wave

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\, E_0 \sin(kz - \omega t)$$

$$\mathbf{H}(z,t) = \hat{\mathbf{y}}\, \sqrt{\frac{\mu_0}{\varepsilon_0}}\, E_0 \sin(kz - \omega t)$$

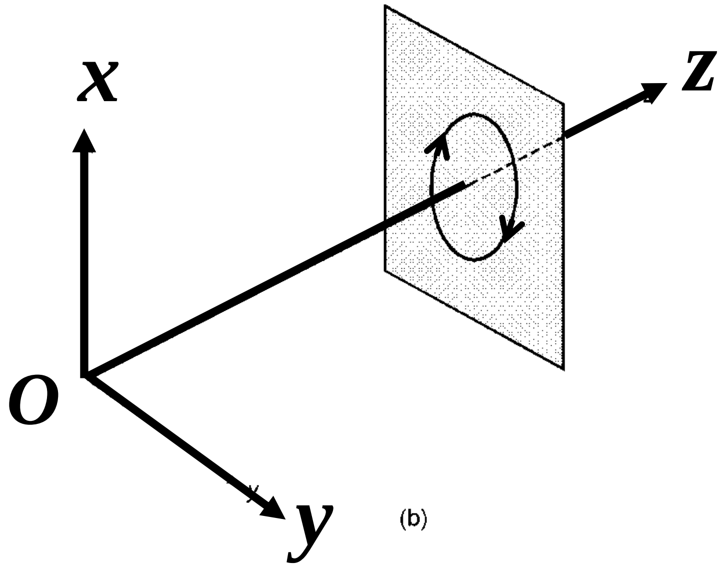


x-polarized wave



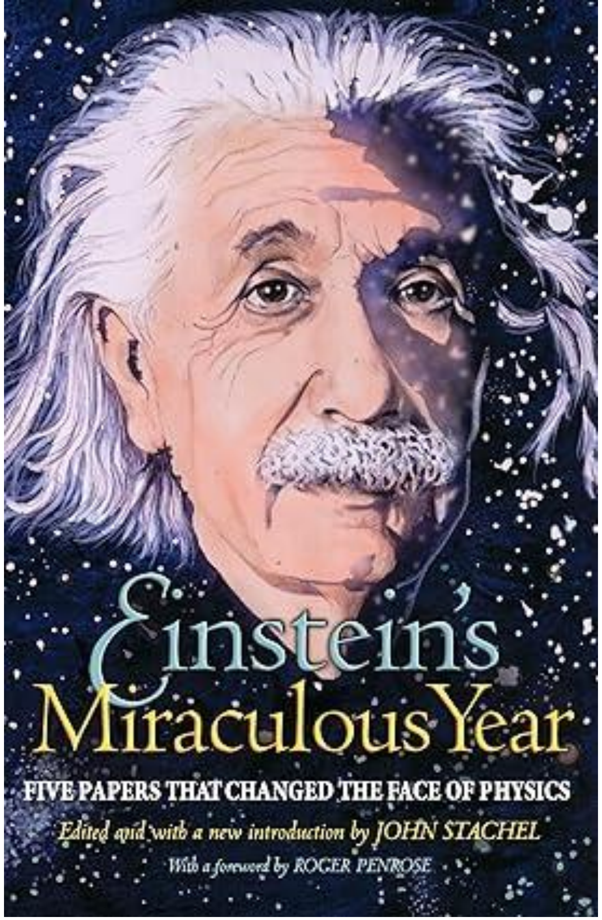
(a)

RCP



(b)

Circularly Polarized Wave



1905: Einstein's Year of Miracles

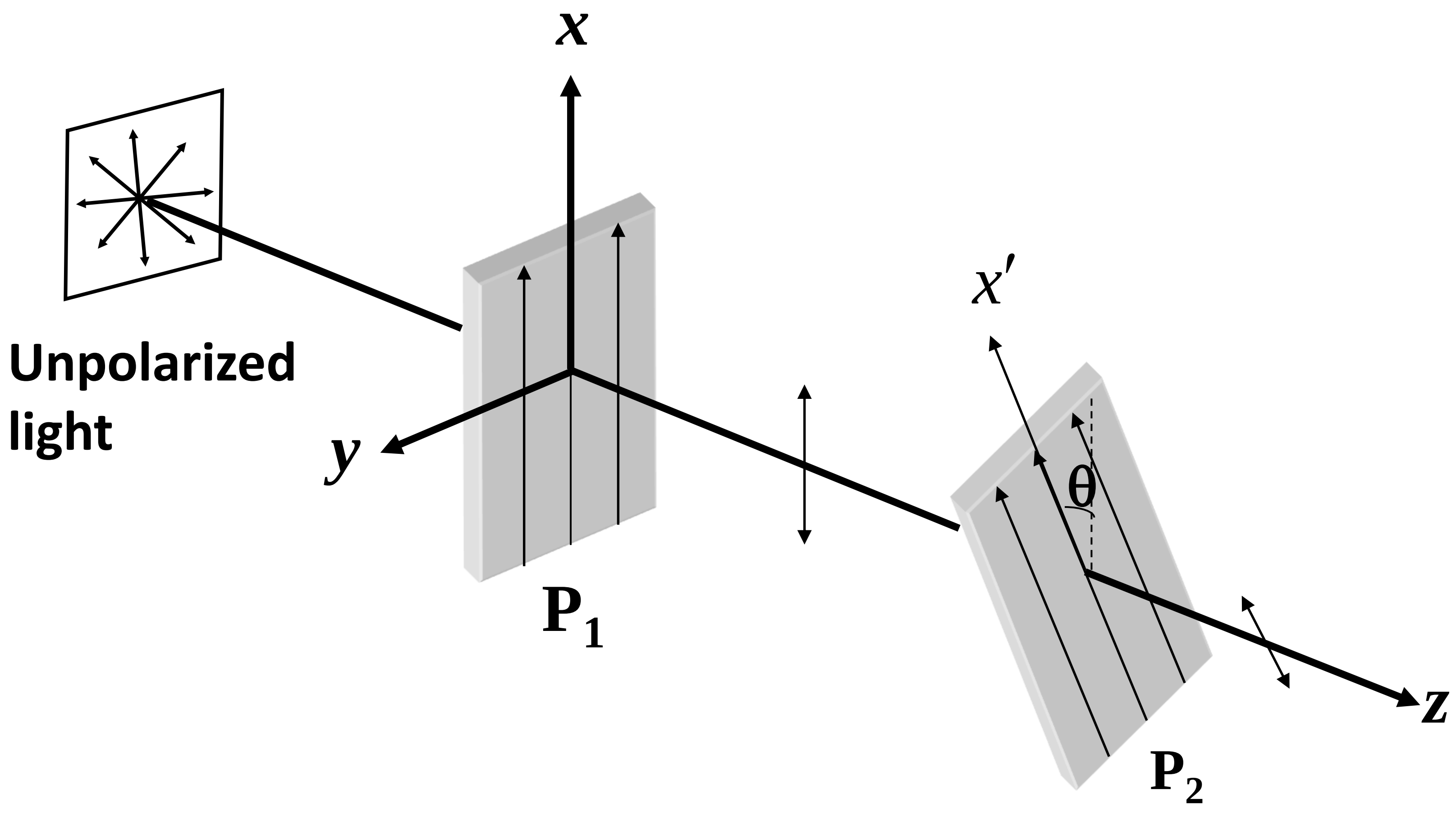
FIVE PAPERS THAT CHANGED THE FACE OF PHYSICS

(Editor John Stachel)

In the second paper in his Year of Miracles, Einstein wrote ***radiation energy consists of indivisible quanta of energy***

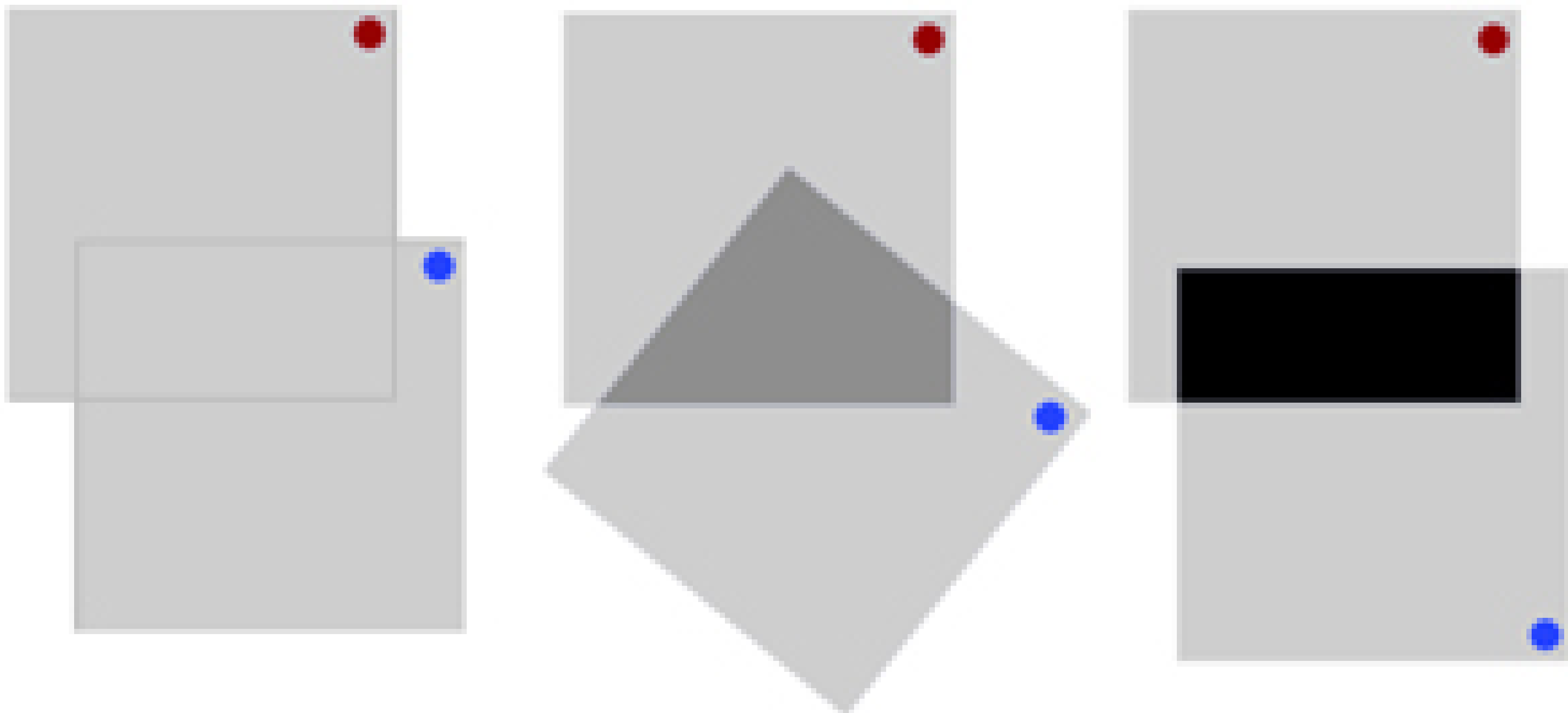
$$E = h\nu \qquad p = \frac{h\nu}{c}$$

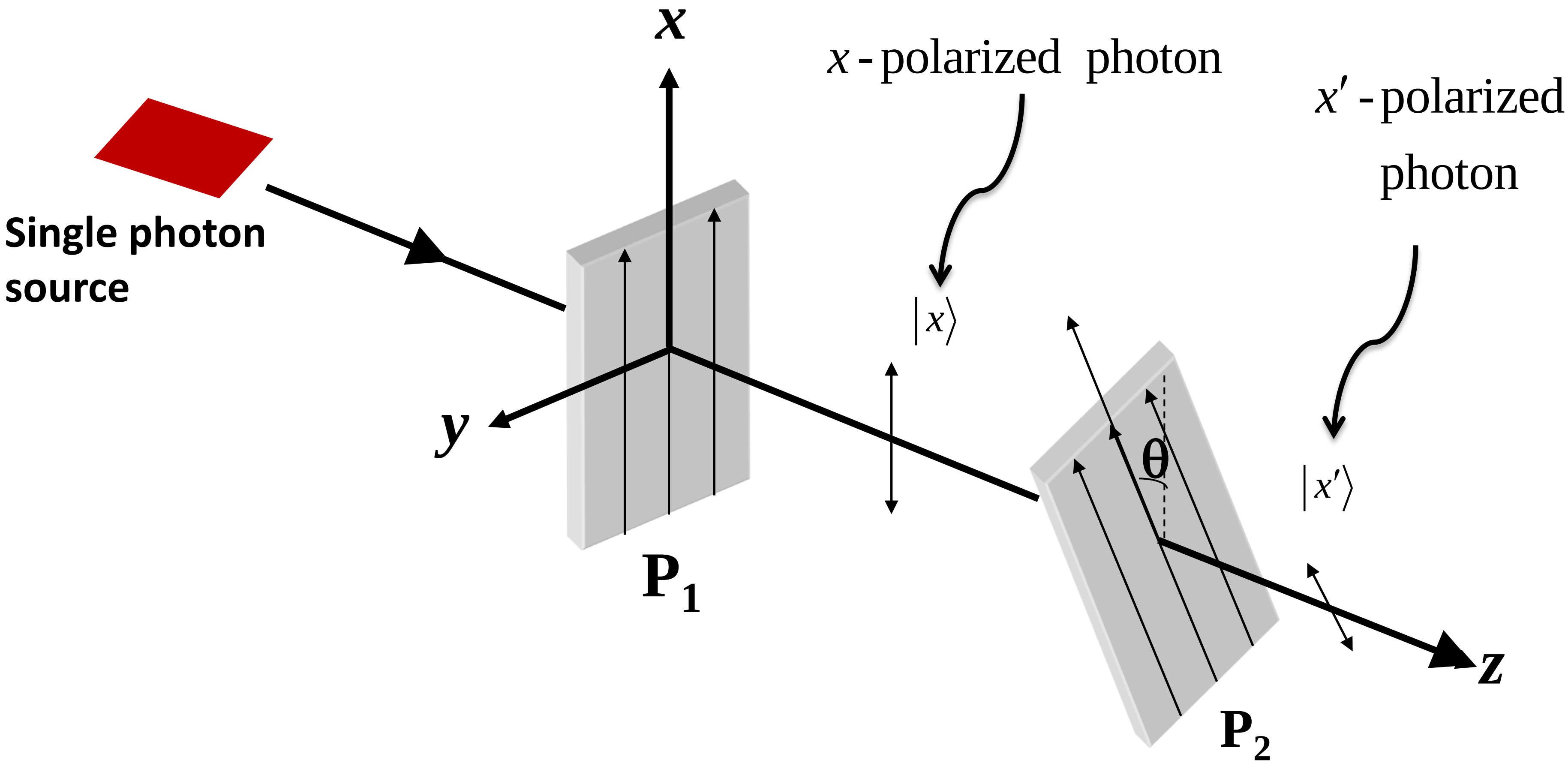
In 1926, Einstein's Light Quantum came to be known as Photon



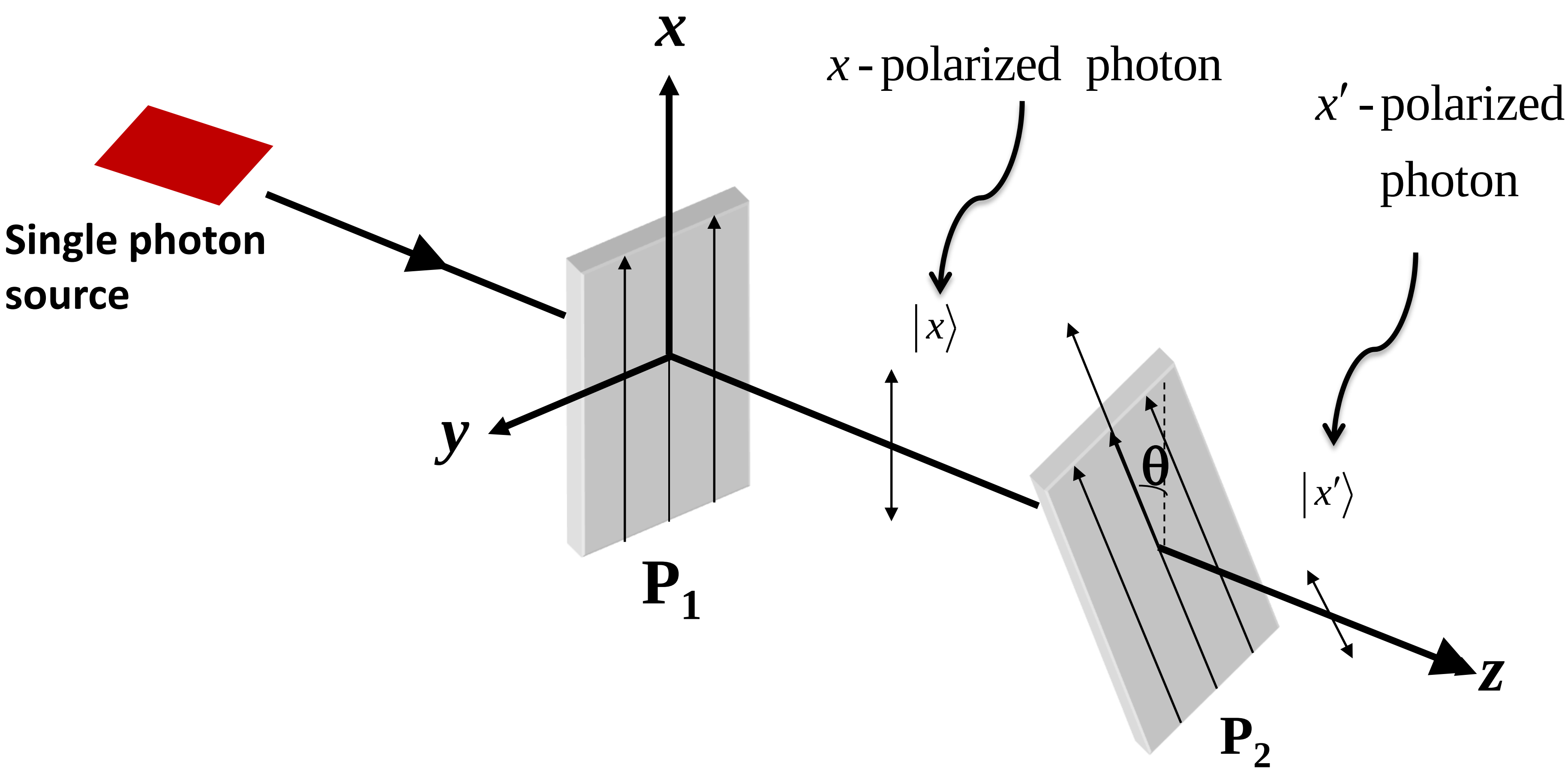
$$E_{x'} = E_x \cos\theta \Rightarrow I = I_0 \cos^2\theta$$

Law of Malus



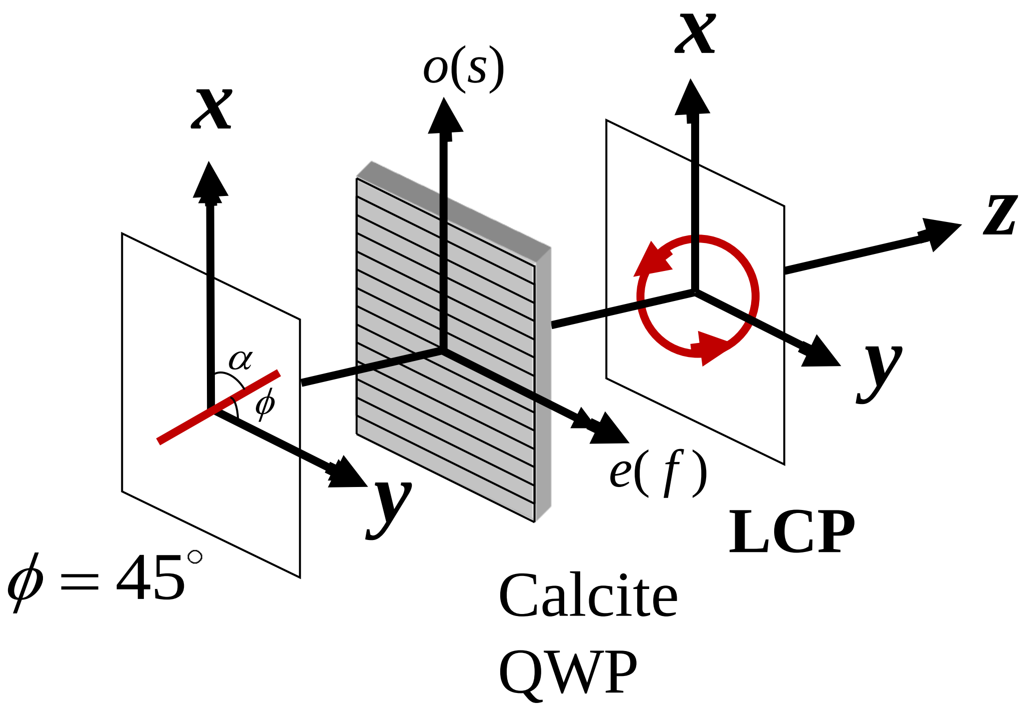


Probability of passing through $= \cos^2 \theta$



Probability of passing through $= \cos^2\theta$

$$|x\rangle = \cos\theta|x'\rangle + \sin\theta|y'\rangle$$



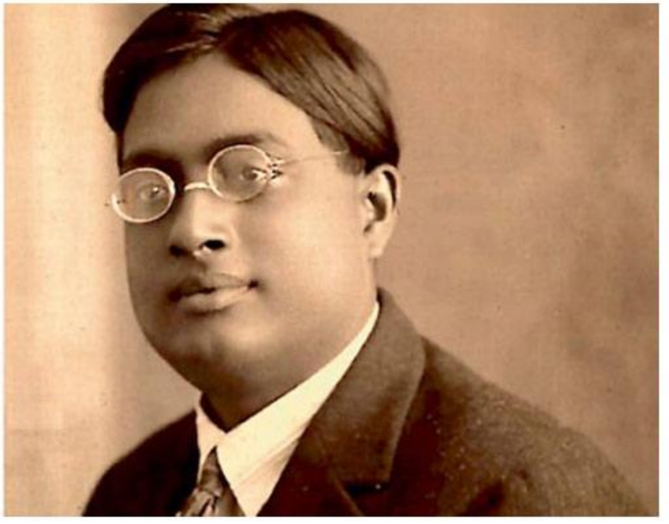
A linearly polarized beam making an angle 45° with the *y*-axis gets converted to a LCP after propagating through a calcite QWP.

$|\text{RCP}\rangle$ photon has an intrinsic angular momentum $+\hbar$

$|\text{LCP}\rangle$ photon has an intrinsic angular momentum $-\hbar$

$|\mathrm{RCP}\rangle$ photon has an intrinsic angular momentum $+\hbar$

$|\mathrm{LCP}\rangle$ photon has an intrinsic angular momentum $-\hbar$



Satyendra Nath Bose

AAPT PHYSICS EDUCATION

American Journal of Physics

LETTERS TO THE EDITOR | AUGUST 01 2024

Who discovered angular momentum of the photon?

Ajoy Ghatak[a)]

Optics and Photonics Center, IIT Delhi, New Delhi, India

(Received 13 December 2023; accepted 28 June 2024)

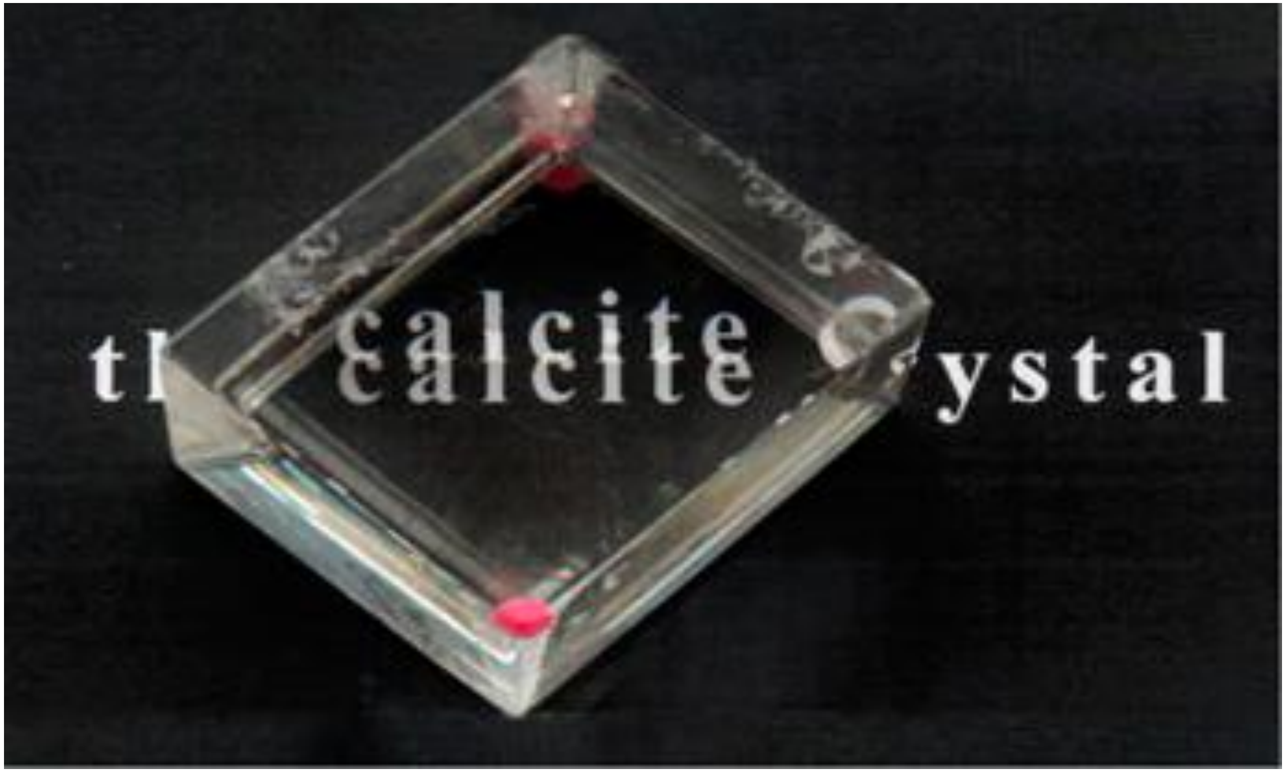
https://doi.org/10.1119/5.0191372

In 1924, Satyendra Nath Bose (then at Dacca University) re-derived Planck's law by developing a new kind of statistics obeyed by light quanta. He asked Albert Einstein to consider the resulting manuscript for publication in *Zeitschrift für Physik*. Einstein translated it into German and published

We may mention
tum of the electr
Uhlenbeck and Gou
Bose had used “
1926 came to be k

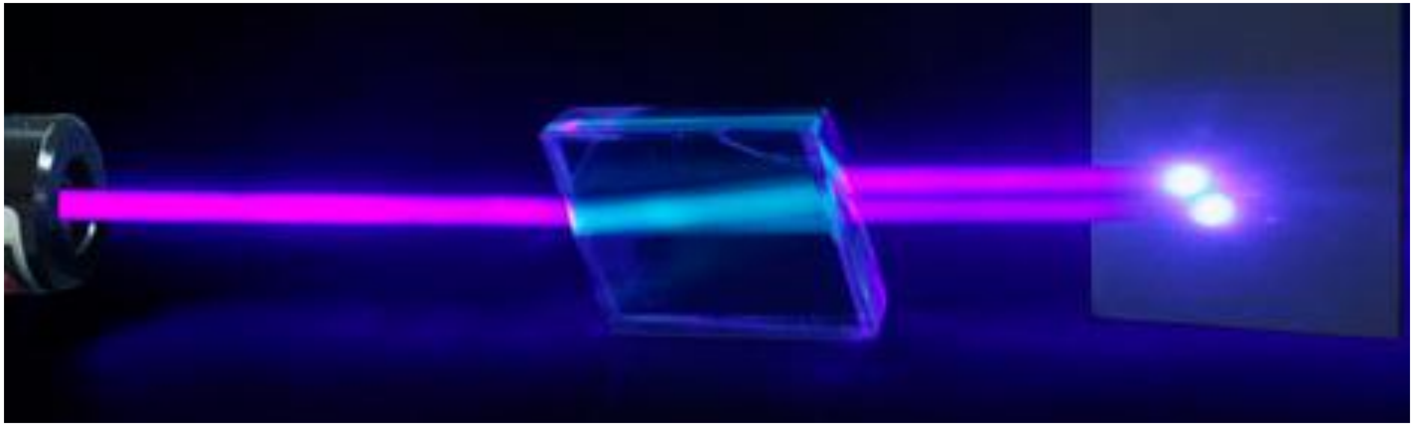
$$\left| x \right\rangle = \frac{1}{\sqrt{2}} \left| \mathrm{RCP} \right\rangle + \frac{1}{\sqrt{2}} \left| \mathrm{LCP} \right\rangle$$

Double refraction in calcite



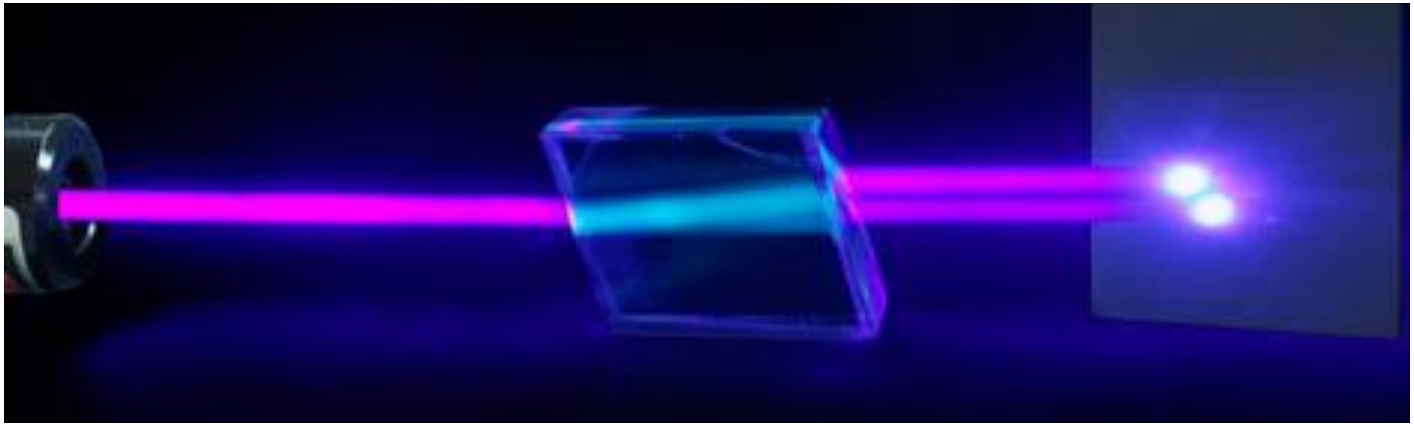
Photograph courtesy Professor V Lakshminarayanan and adapted from G. Ropars, A. Le Flocha and V. Lakshminarayanan, *The sunstone and polarised skylight: ancient Viking navigational tools* ?, Contemporary Physics (2014).

Double refraction in calcite



When an unpolarized light beam is incident normally on a calcite crystal, it usually splits up into two linearly polarized beams.

Double refraction in calcite



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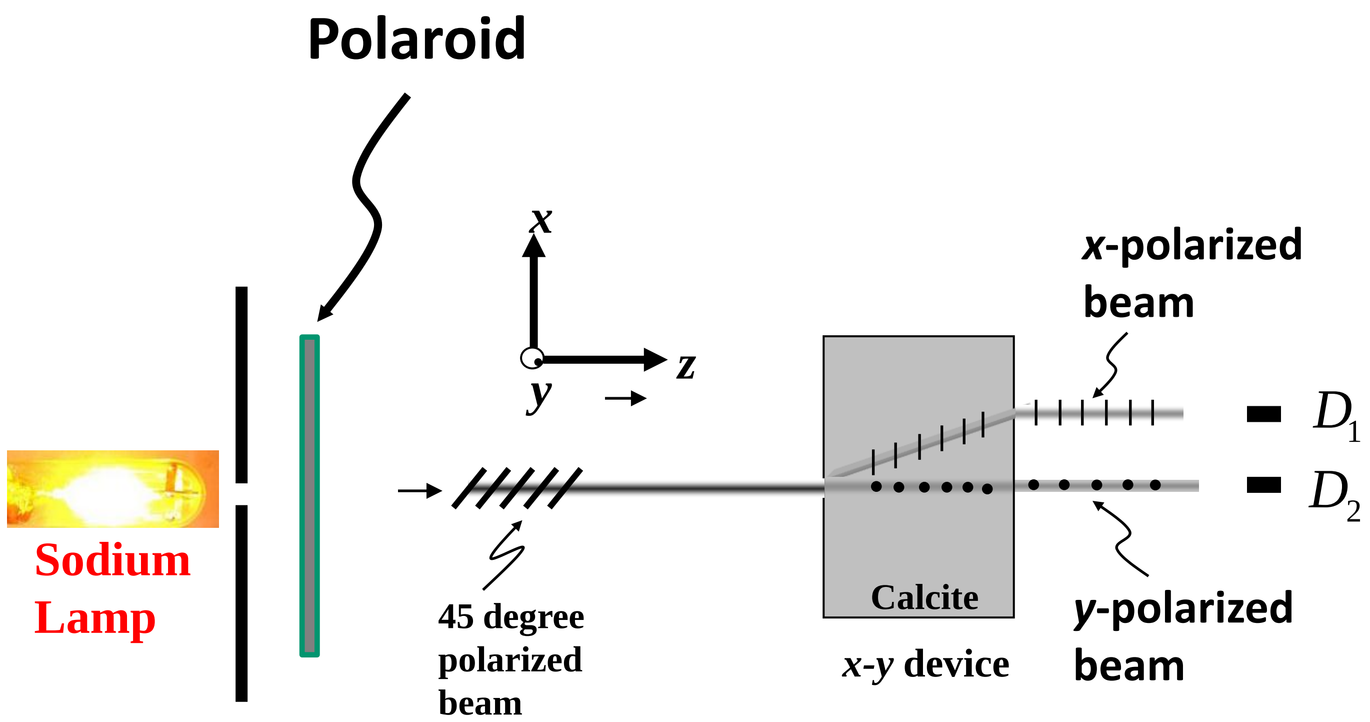
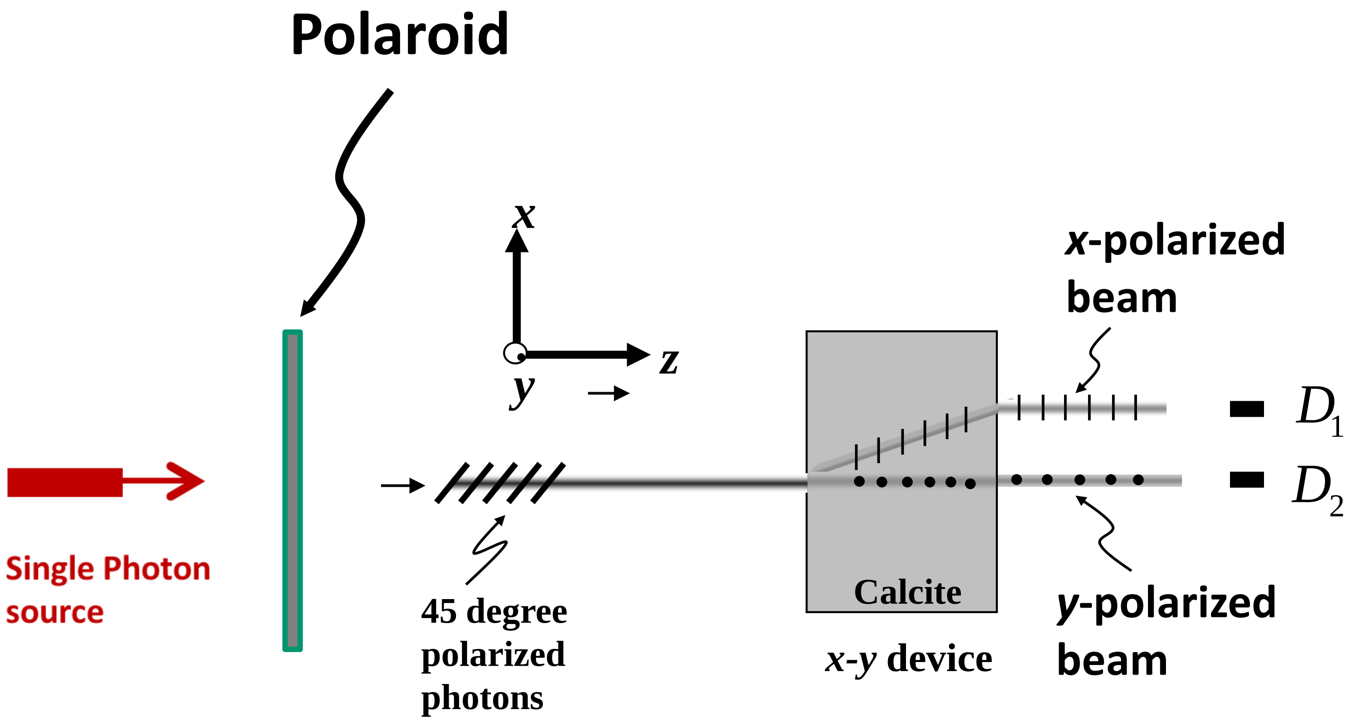
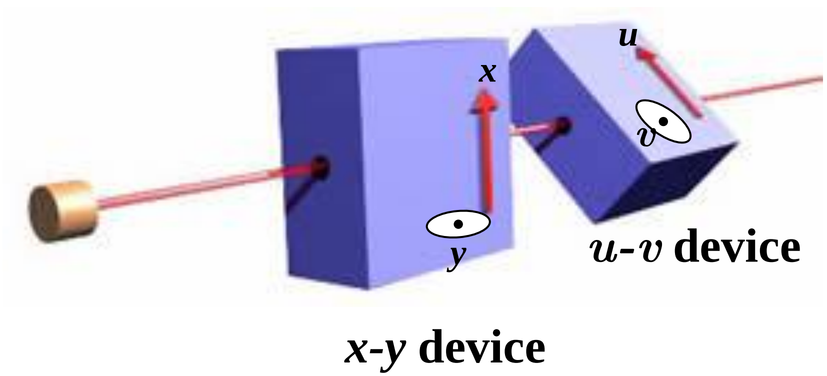
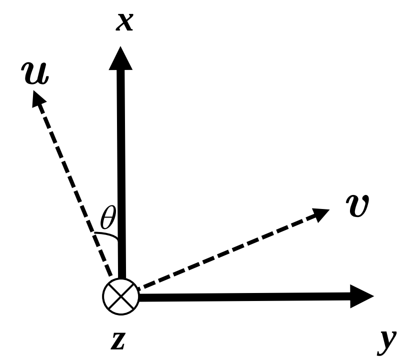

Polaroid
x
z
y
x-polarized beam
D_1
D_2
Sodium Lamp
45 degree polarized beam
Calcite
x-y device
y-polarized beam

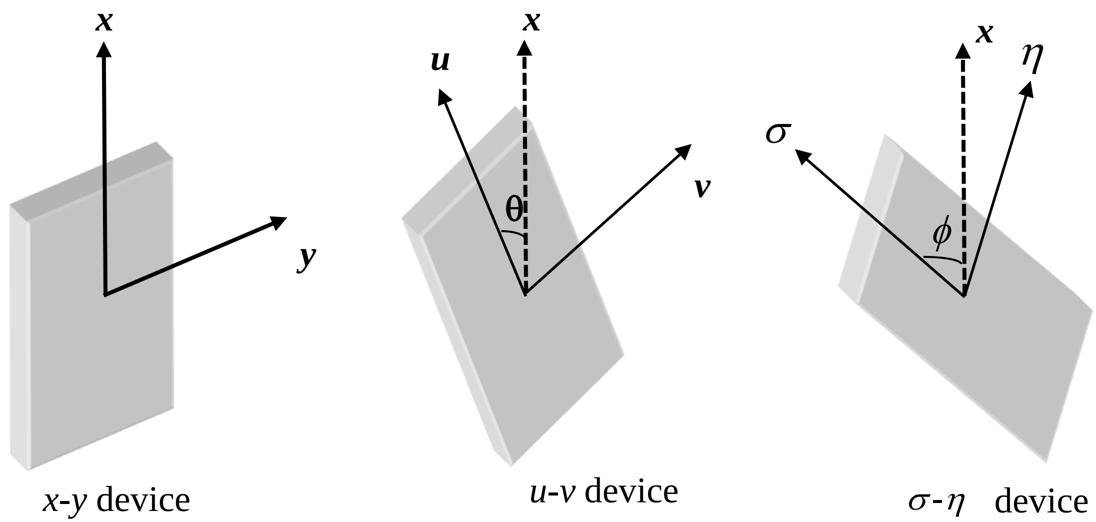


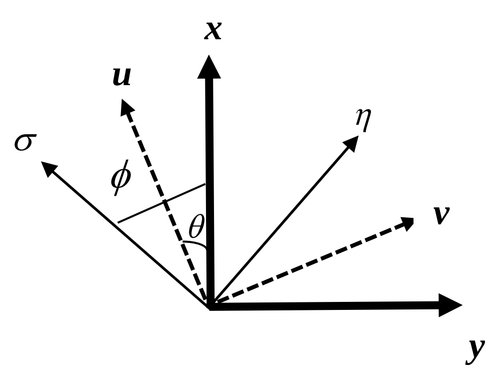
$$|45^\circ\rangle = \frac{1}{\sqrt{2}}|x\rangle + \frac{1}{\sqrt{2}}|y\rangle$$

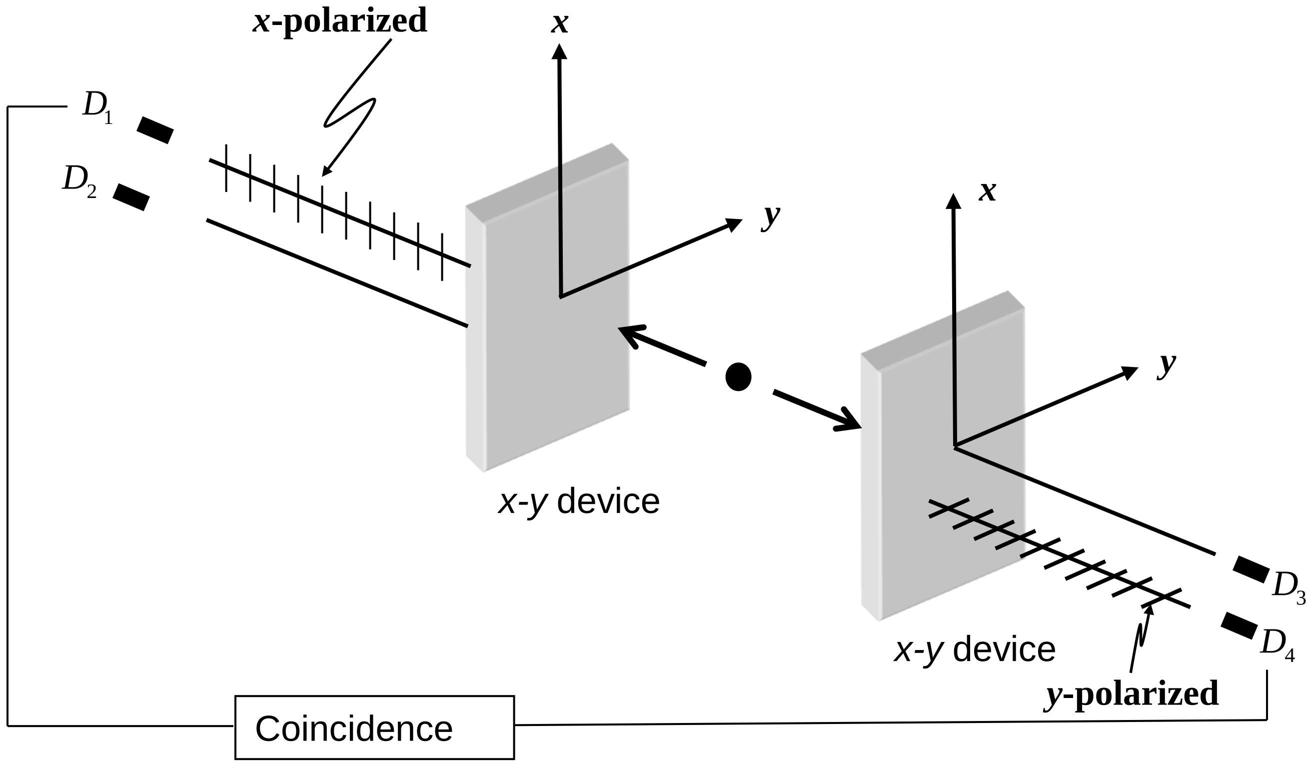




Adapted from
http://www.upscale.utoronto.ca/PVB/Harrison/BellsTheorem/BellsTheorem.html


x
y
x-y device
x
u
v
θ
u-v device
x
η
σ
ϕ
σ-η device


x
u
σ
η
ϕ
θ
v
y



x-polarized
x
y
x
y
D1
D2
D3
D4
x-y device
x-y device
y-polarized
Coincidence

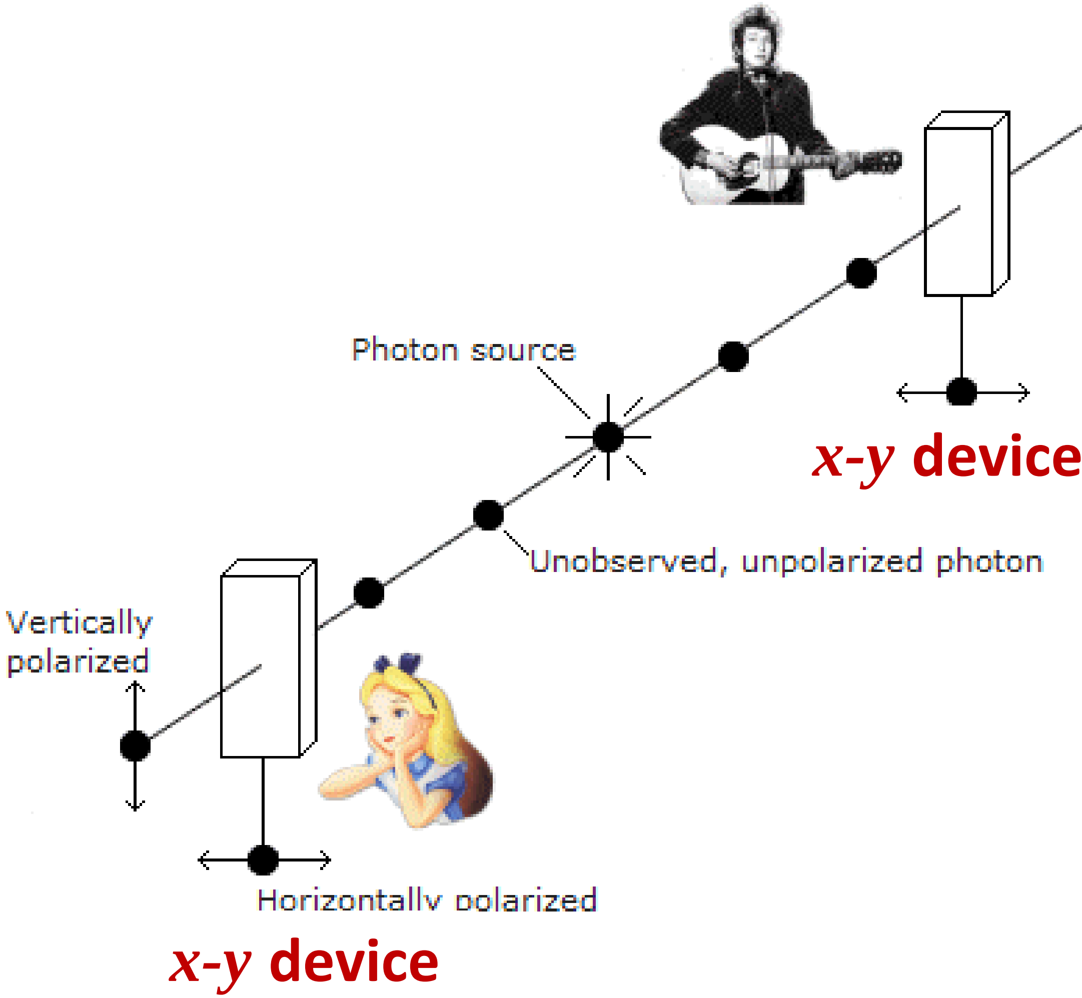
According to quantum theory:

the polarization of the photon

(traveling to the left – or to the right)

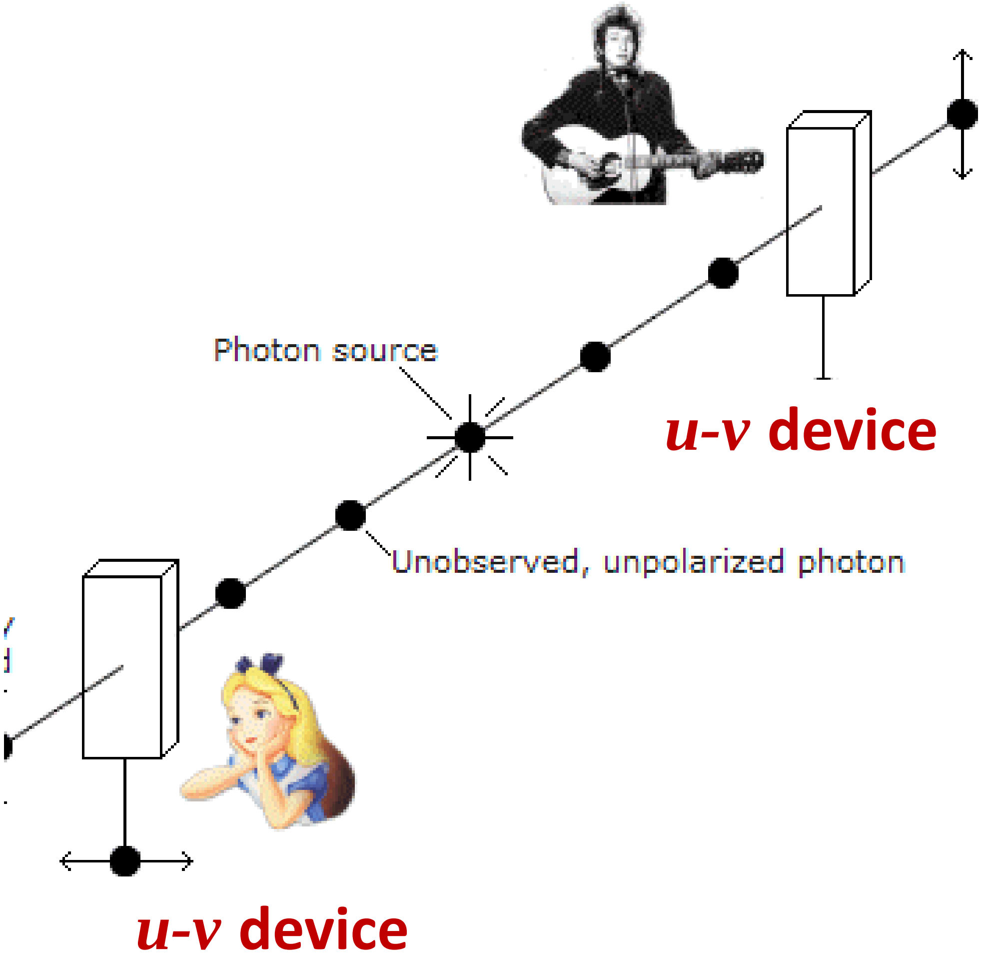
is not known before one of them is

measured;



If the photon going to the left is passed through an *x*-*y* device, and it is found to be *x*-polarized then the photon going to the right collapses to a state in which its polarization state is *y*-polarized

Diagram adapted from
http://www.faculty.umb.edu/gary_zabel/Courses/Parallel%20Universes/Texts/Quantum%20Entanglement.htm
(Based on a diagram from the book *Quantum Enigma*)



Similarly, if the photon going to the left is passed through an *u*-*v* device, and it is found to be *u*- polarized then the polarization of the photon going to the right collapses to a state in which its polarization state is for sure *v* - polarized.

Diagram adapted from
http://www.faculty.umb.edu/gary_zabel/Courses/Parallel%20Universes/Texts/Quantum%20Entanglement.htm
(Based on a diagram from the book *Quantum Enigma*)

Whatever happened to one particle would thus immediately affect the other particle, wherever in the universe it may be. Einstein called this

Spooky action at a distance

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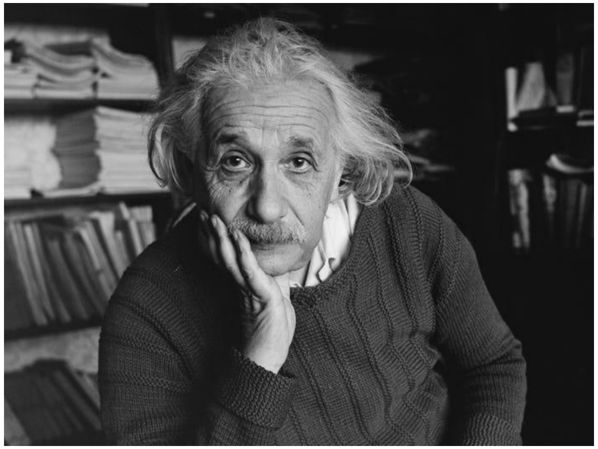


In German, Einstein said

Spukhafte Fernwirkung

In 1935 Einstein, Podolsky & Rosen wrote in a paper

If quantum theory was correct, then two particles (which are millions of kilometers apart) can be entangled in the sense that by determining a property of one of the particles, the property of the second particle can be instantaneously changed.

And special theory of relativity forbids the transmission of any signal faster than the speed of light.

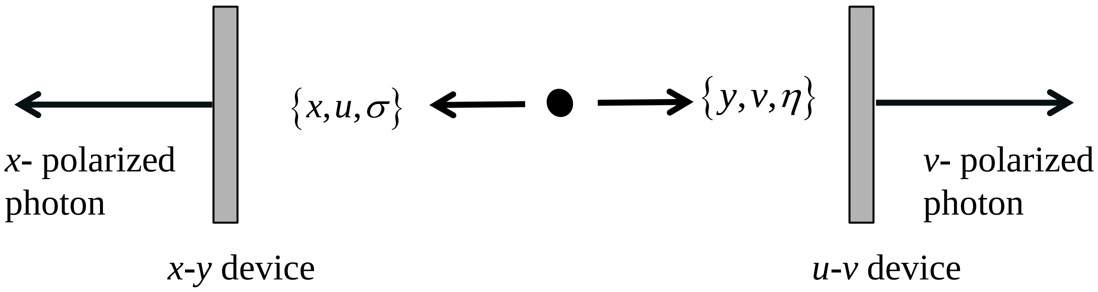
This came to be known as *The EPR Paradox*.

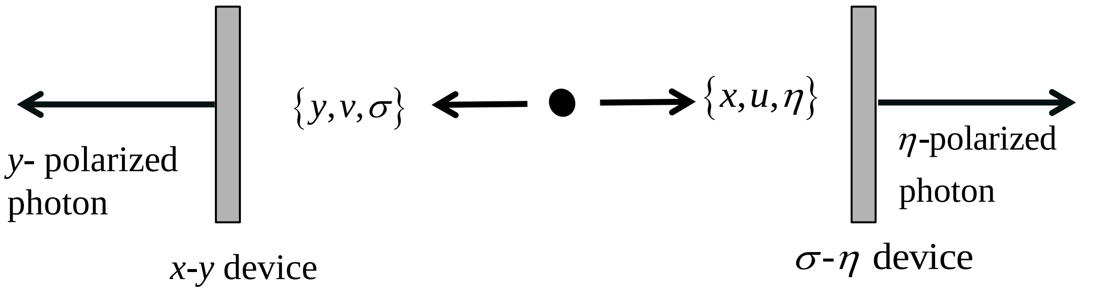
About thirty years later, experiments confirmed the predictions of quantum mechanics namely Einstein's impossible proposition was in fact correct:

instantaneous changes in widely separated systems did occur

EPR argued that, if without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then the second particle must have possessed the measured property before the measurement was carried out.

We assume that the photon must carry all the information; thus there must exist "hidden variables" so that we can assign values of all observables of a system.





	Photons going to the left	Photons going to the right
N_1 pairs characterized by	$\{x,u,\sigma\}$	$\{y,v,\eta\}$
N_2 pairs characterized by	$\{x,u,\eta\}$	$\{y,v,\sigma\}$
N_3 pairs characterized by	$\{x,v,\sigma\}$	$\{y,u,\eta\}$
N_4 pairs characterized by	$\{x,v,\eta\}$	$\{y,u,\sigma\}$
N_5 pairs characterized by	$\{y,u,\sigma\}$	$\{x,v,\eta\}$
N_6 pairs characterized by	$\{y,u,\eta\}$	$\{x,v,\sigma\}$
N_7 pairs characterized by	$\{y,v,\sigma\}$	$\{x,u,\eta\}$
N_8 pairs characterized by	$\{y,v,\eta\}$	$\{x,u,\sigma\}$

Let us calculate $P(x,u)$

	Photons going to the left	Photons going to the right	
N_1 pairs characterized by	$\{x, u, \sigma\}$	$\{y, v, \eta\}$	
N_2 pairs characterized by	$\{x, u, \eta\}$	$\{y, v, \sigma\}$	
N_3 pairs characterized by	$\{x, v, \sigma\}$	$\{y, u, \eta\}$	←
N_4 pairs characterized by	$\{x, v, \eta\}$	$\{y, u, \sigma\}$	←
N_5 pairs characterized by	$\{y, u, \sigma\}$	$\{x, v, \eta\}$	
N_6 pairs characterized by	$\{y, u, \eta\}$	$\{x, v, \sigma\}$	
N_7 pairs characterized by	$\{y, v, \sigma\}$	$\{x, u, \eta\}$	
N_8 pairs characterized by	$\{y, v, \eta\}$	$\{x, u, \sigma\}$	

	Photons going to the left	Photons going to the right
$P(x,u)$		
N_3 pairs characterized by	$\{x, v, \sigma\}$	$\{y, u, \eta\}$ ←
N_4 pairs characterized by	$\{x, v, \eta\}$	$\{y, u, \sigma\}$ ←

$$P(x,u) = \frac{N_3 + N_4}{N}$$

$$N = N_1 + N_2 + N_3 + N_4 + N_5 + N_6 + N_7 + N_8$$

We next calculate $P(x, \sigma)$

	Photons going to the left	Photons going to the right
N_1 pairs characterized by	$\{x,u,\sigma\}$	$\{y,v,\eta\}$
N_2 pairs characterized by	$\{x,u,\eta\}$	$\{y,v,\sigma\}$ ←
N_3 pairs characterized by	$\{x,v,\sigma\}$	$\{y,u,\eta\}$
N_4 pairs characterized by	$\{x,v,\eta\}$	$\{y,u,\sigma\}$ ←
N_5 pairs characterized by	$\{y,u,\sigma\}$	$\{x,v,\eta\}$
N_6 pairs characterized by	$\{y,u,\eta\}$	$\{x,v,\sigma\}$
N_7 pairs characterized by	$\{y,v,\sigma\}$	$\{x,u,\eta\}$
N_8 pairs characterized by	$\{y,v,\eta\}$	$\{x,u,\sigma\}$

	Photons going to the left	Photons going to the right	
N_1 pairs characterized by	$\{x,u,\sigma\}$	$\{y,v,\eta\}$	
N_2 pairs characterized by	$\{x,u,\eta\}$	$\{y,v,\sigma\}$	←
N_3 pairs characterized by	$\{x,v,\sigma\}$	$\{y,u,\eta\}$	
N_4 pairs characterized by	$\{x,v,\eta\}$	$\{y,u,\sigma\}$	←
N_5 pairs characterized by	$\{y,u,\sigma\}$	$\{x,v,\eta\}$	
N_6 pairs characterized by	$\{y,u,\eta\}$	$\{x,v,\sigma\}$	
N_7 pairs characterized by	$\{y,v,\sigma\}$	$\{x,u,\eta\}$	
N_8 pairs characterized by	$\{y,v,\eta\}$	$\{x,u,\sigma\}$	

$$P(x,\sigma) = \frac{N_2 + N_4}{N}$$

$$N = N_1 + N_2 + N_3 + N_4 + N_5 + N_6 + N_7 + N_8$$

$P(\sigma, u)$

	Photons going to the left	Photons going to the right
N_1 pairs characterized by	$\{x, u, \sigma\}$	$\{y, v, \eta\}$
N_2 pairs characterized by	$\{x, u, \eta\}$	$\{y, v, \sigma\}$
N_3 pairs characterized by	$\{x, v, \sigma\}$	$\{y, u, \eta\}$ ←
N_4 pairs characterized by	$\{x, v, \eta\}$	$\{y, u, \sigma\}$
N_7 pairs characterized by	$\{y, v, \sigma\}$	$\{x, u, \eta\}$ ←

$$P(\sigma, u) = \frac{N_3 + N_7}{N}$$

$$N = N_1 + N_2 + N_3 + N_4 + N_5 + N_6 + N_7 + N_8$$

$$P(x,\sigma) = \frac{N_2 + N_4}{N}$$

$$P(\sigma,u) = \frac{N_3 + N_7}{N}$$

$$P(x,\sigma) + P(\sigma,u) = \frac{N_2 + N_4 + N_3 + N_7}{N}$$

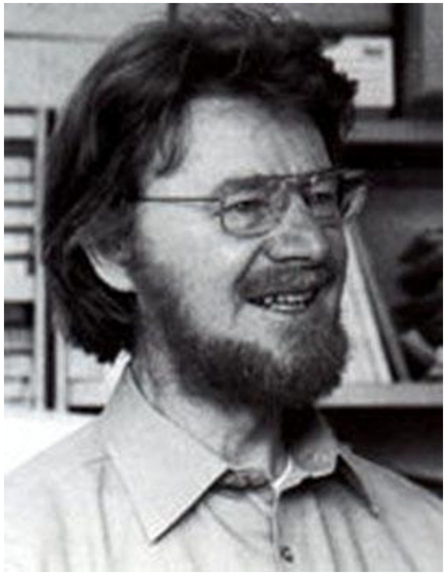
$$P(x,\sigma) + P(\sigma,u) = \frac{N_2 + N_4 + N_3 + N_7}{N}$$

$$P(x,u) = \frac{N_3 + N_4}{N}$$

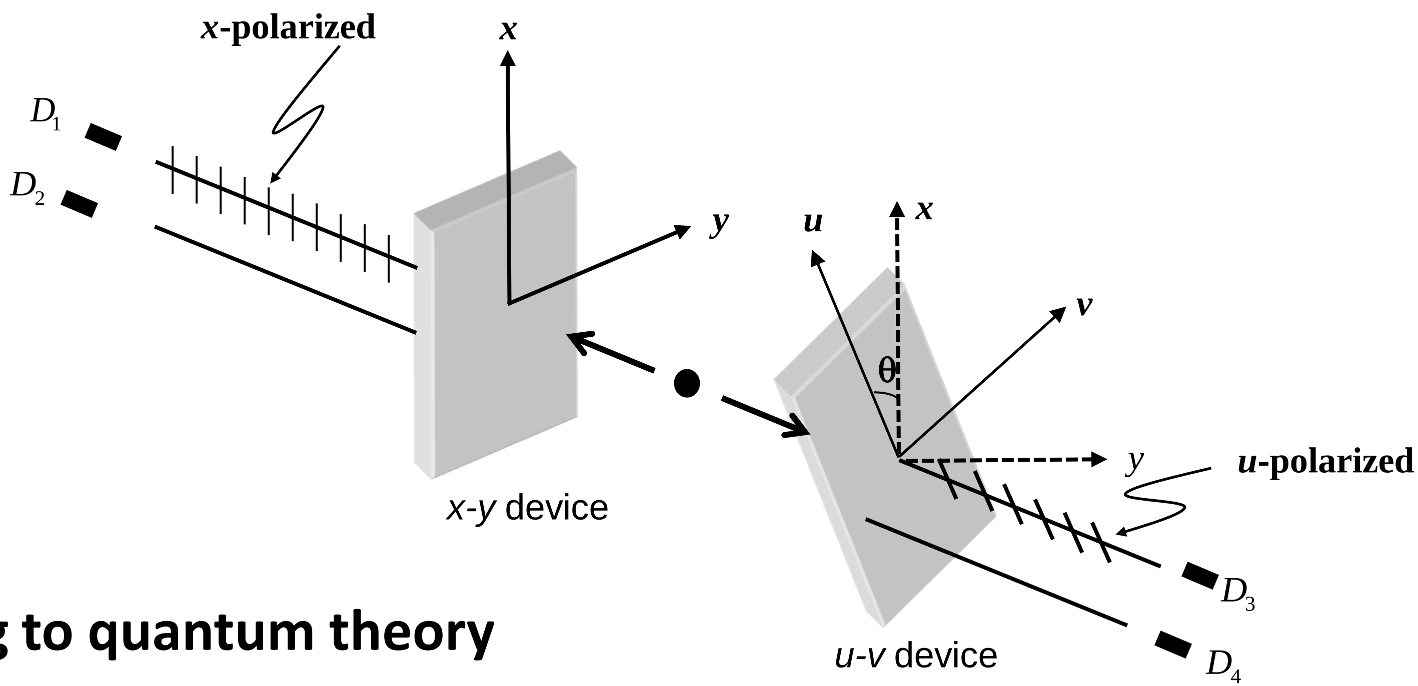
$$P(x,u) \leq P(x,\sigma) + P(\sigma,u)$$

$$P(x,u) \leq P(x,\sigma) + P(\sigma,u)$$

This is a simple form of Bell's inequality



John Stewart Bell (1928 – 1990) was an outstanding Irish physicist, and the originator of Bell's Theorem, one of the most important theorems in quantum physics.

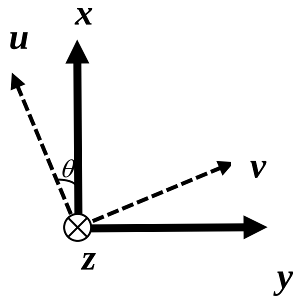


According to quantum theory

$$P\left(x,u\right)=\frac{1}{2}\cos^2\left(\frac{\pi}{2}+\theta\right)=\frac{1}{2}\sin^2\theta$$

Now, according to Quantum Mechanics

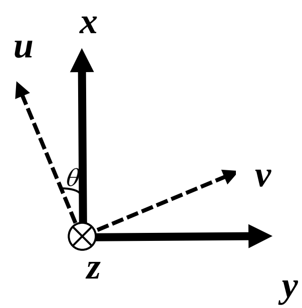
$$P(x,u) = \frac{1}{2}\sin^2\theta$$



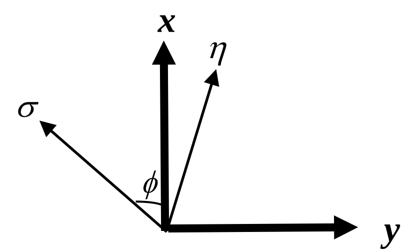
u-v device

Now, according to Quantum Mechanics

$$P(x,u) = \frac{1}{2}\sin^2\theta \qquad P(x,\sigma) = \frac{1}{2}\sin^2\phi \qquad P(\sigma,u) = \frac{1}{2}\sin^2(\phi-\theta)$$



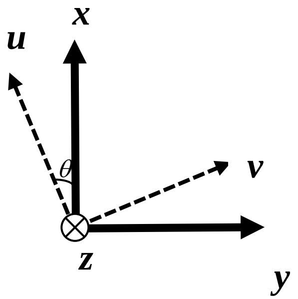
u-v device



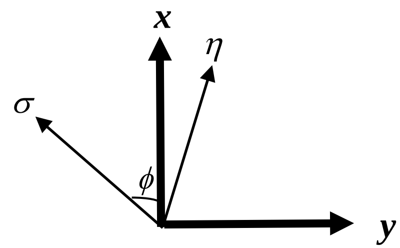
σ-η device

Now, according to Quantum Mechanics

$$P(x,u)=\frac{1}{2}\sin^2\theta \qquad P(x,\sigma)=\frac{1}{2}\sin^2\phi \qquad P(\sigma,u)=\frac{1}{2}\sin^2(\phi-\theta)$$



u-v device



σ-η device

Therefore Bell's inequality

$$P(x,u)\le P(x,\sigma)+P(\sigma,u)$$

will imply

$$\sin^2\theta\le\sin^2\phi+\sin^2(\phi-\theta)$$

Thus, according to Quantum Theory, Bell's inequality will imply

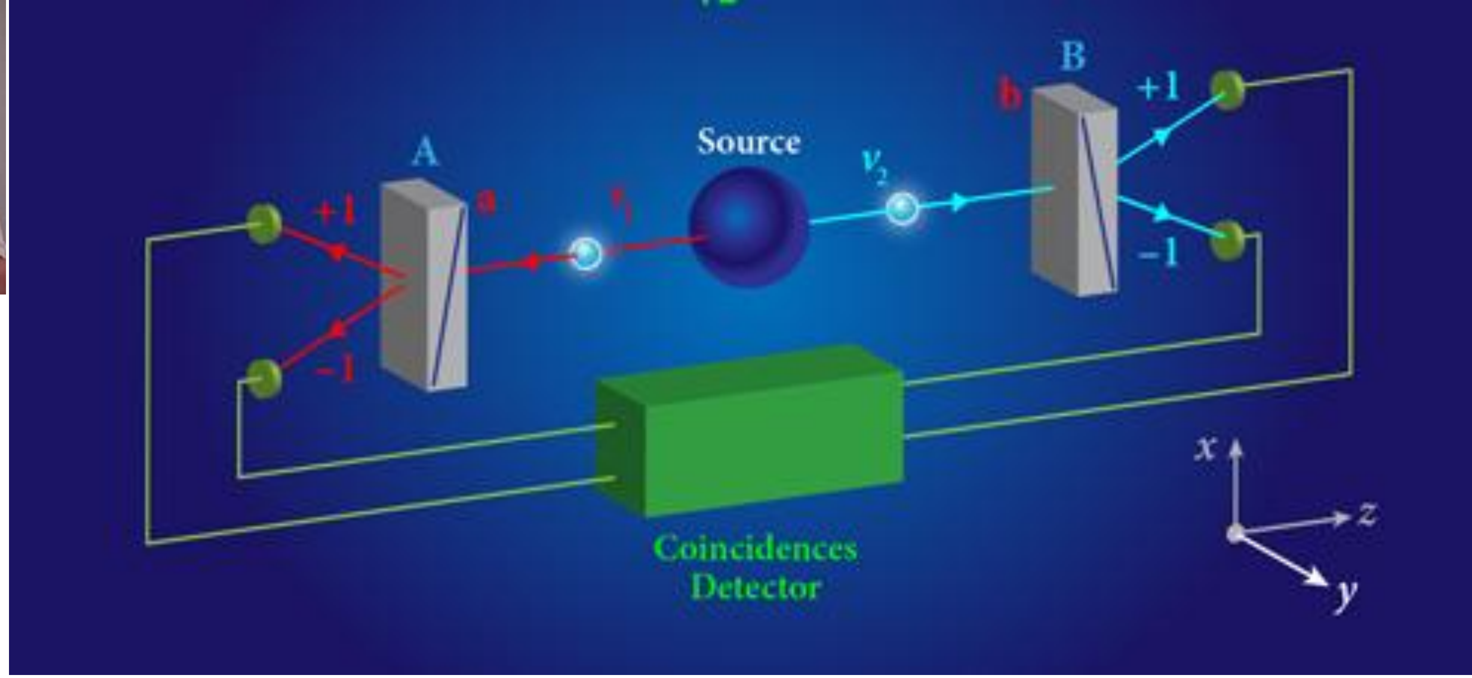
$$\sin^2\theta \le \sin^2\phi + \sin^2(\phi-\theta)$$

If we assume $\theta = 2\phi,$ **the above inequality will imply**

$\sin^2 2\phi \le 2\sin^2\phi$ for $\phi = \frac{\pi}{6},$ we will have

$$\frac{3}{4} \le \frac{2}{4} \quad \Rightarrow 0.75 \le 0.5$$

Thus quantum theory and Bell's inequality are not compatible which implies that either quantum theory is right or theory based on hidden variables, but not both.



An apparatus for performing a Bell test. A source emits a pair of entangled photons v1 and v2. Their polarizations are analyzed by polarizers A and B (grey blocks), which are aligned, respectively, along directions *a* and *b* (*a* and *b* can be along *x*, *y* or any direction in the *x*-*y* plane; here, they are along *x*.)

Adapted from:Alain Aspect, *Physics* 8, 123, Dec 16, 2015.

Aspect etal write:

The linear-polarization correlation of pairs of photons emitted in a radiative cascade of calcium has been measured. The new experimental scheme, using two-channel polarizers is a straightforward transposition of EPR *gedanken experiment*. The present results, in excellent agreement with the quantum mechanical predictions, lead to the greatest violation of Bell's inequalities ever achieved.

Bell’s theorem represents (to quote Anton Zeilinger)

…as one of the most profound discoveries since Copernicus… Bell delivered a death blow to the local realistic picture of the world. .. many experiments have demonstrated that the predictions of quantum mechanics for entangled particles are fully correct … and the world is really as “crazy” as predicted by quantum mechanics.

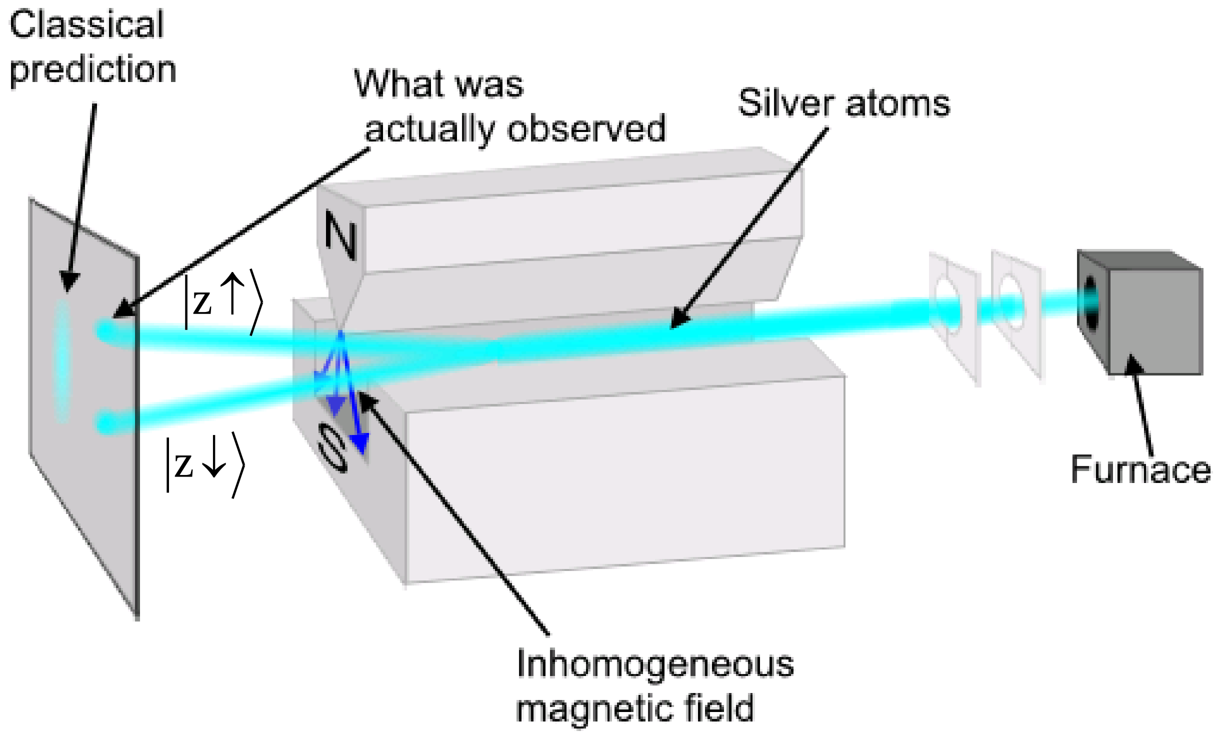
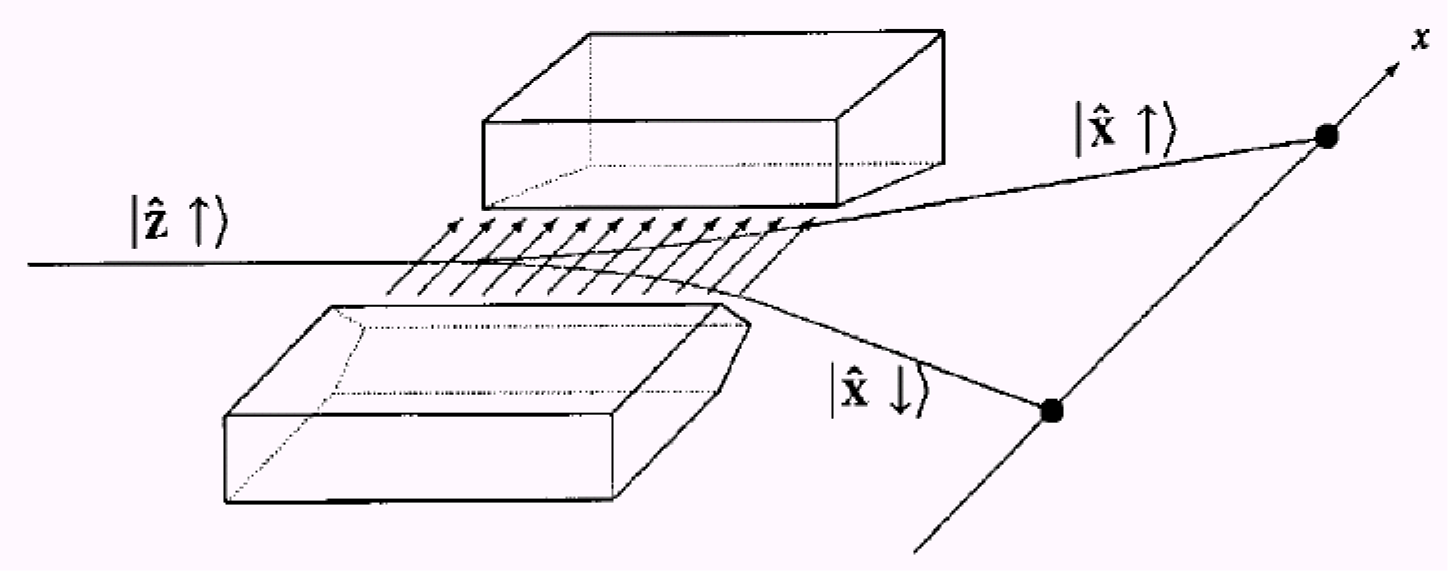


Diagram drawn by en wikipedia Theresa Knott.
http://en.wikipedia.org/wiki/Stern_gerlach_experiment

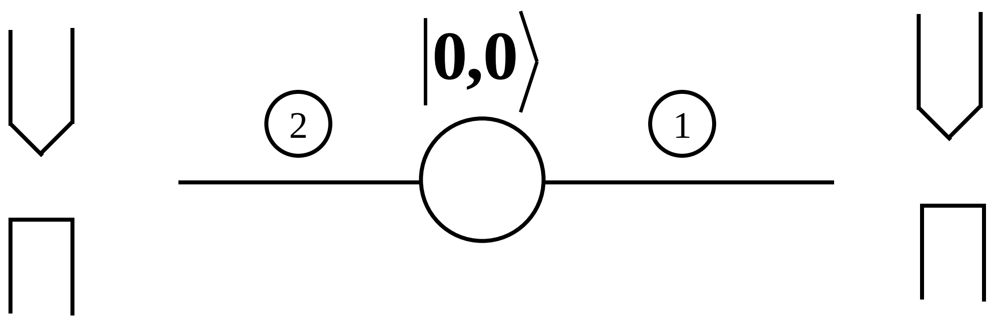
$$|\mathbf{z}\uparrow\rangle = \frac{1}{\sqrt{2}}|\mathbf{x}\uparrow\rangle + \frac{1}{\sqrt{2}}|\mathbf{x}\downarrow\rangle$$



Assume 2 spin half particles in the singlet state

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right]$$

Suppose at $t = 0$ it dissociates and the two particles go off in opposite directions

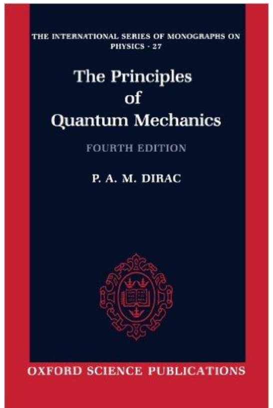


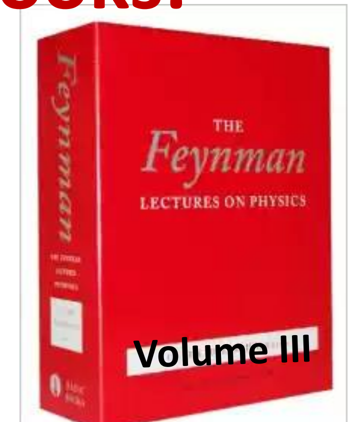
The most fundamental theory now available is probabilistic in form, and not deterministic ..

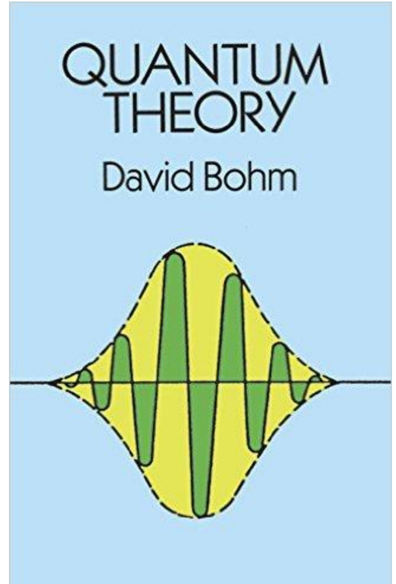
...David Bohm

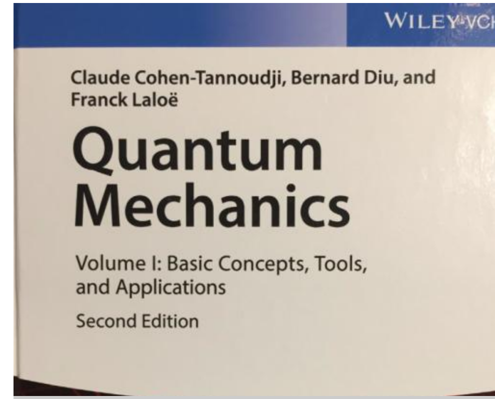
David Bohm was widely considered one of the best quantum physicists of all time.

Reference books:







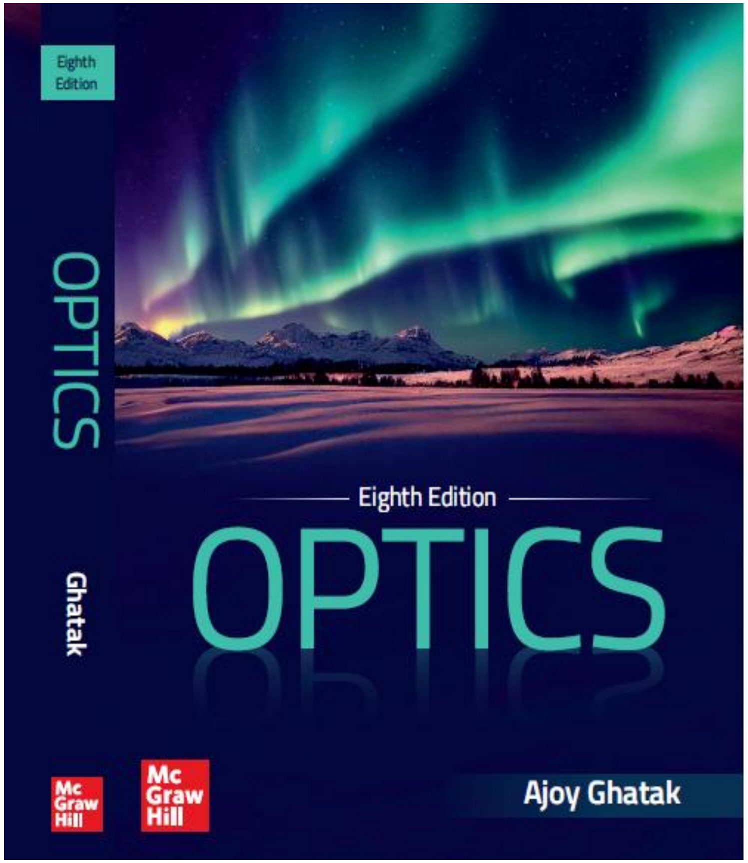


Translated by
Susan Hemley
Nicole Ostrowsky
& Dan Ostrowsky

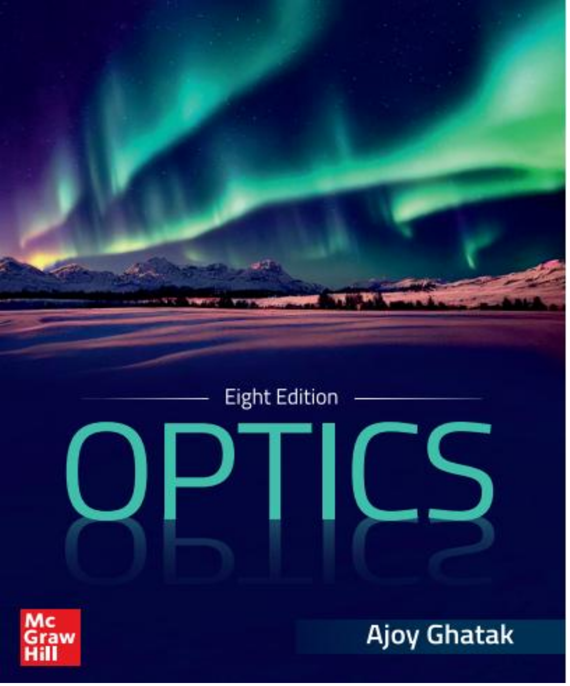
Dirac wanted to understand physics by constructing mathematics, while Feynman started from what he observed in the real world.

Photo by Marek Holzman

Whatever I have said today is given in

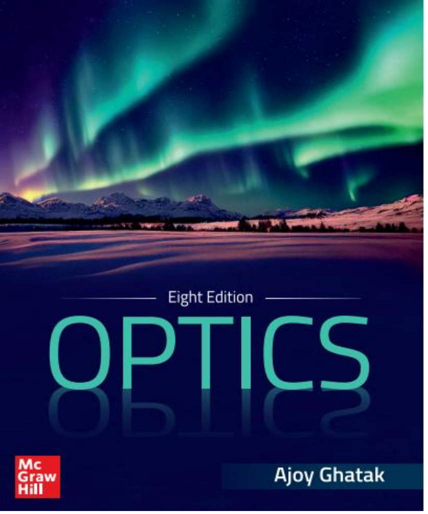


OPTICS, 8th Edition (2024)
Ajoy Ghatak,
McGraw-Hill Education (India)



Many thanks for your attention

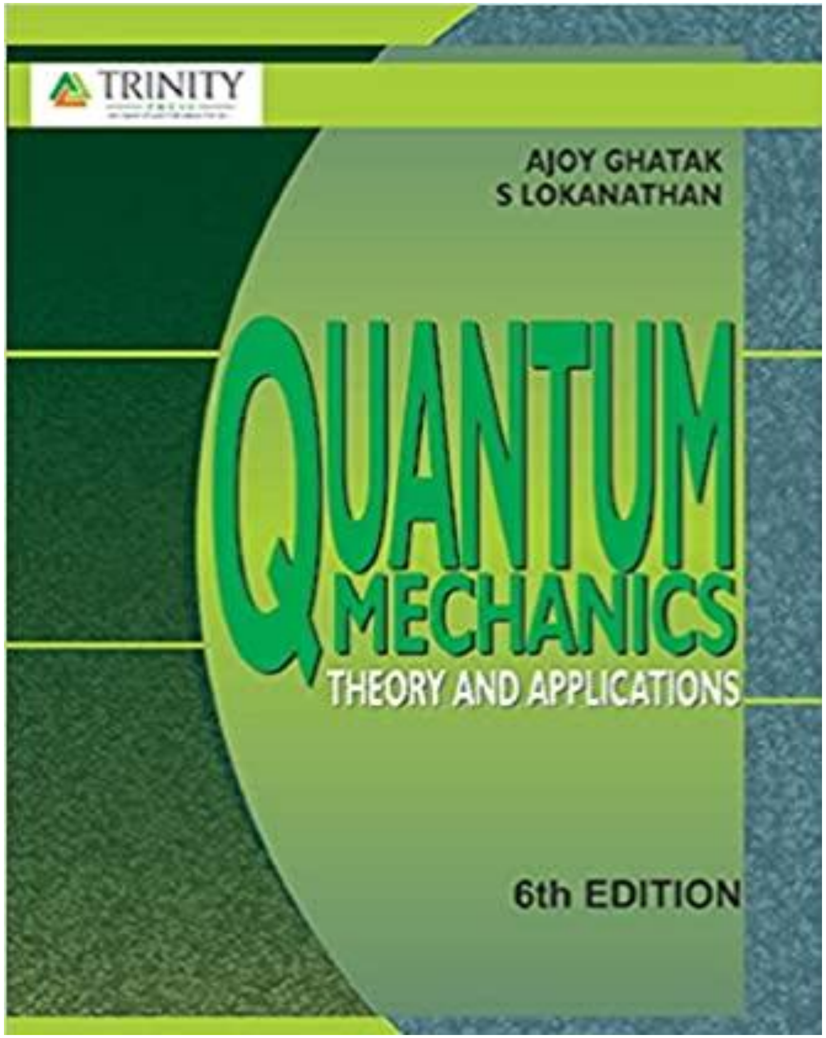
Many thanks for your attention



The spectacular display of colors in the northern and in the southern hemispheres are called "Aurora Borealis"

The Aurora can be seen near the north and south poles; The colors are caused by the following process: charged particles (emitted by the sun and trapped in the magnetic field near the poles) collide with the atoms and molecules in our atmosphere and the excited atoms and molecules emit light which result in the beautiful display of colors. The green color is primarily due to excitation of oxygen atoms while the purple or pink are caused by excitation of nitrogen atoms.

I have discussed the Stern Gerlach experiment in great detail in this book



- **QUANTUM MECHANICS: Theory & Applications**, (2019), **6th Edition**, Ajoy Ghatak and S. Lokanathan, Laxmi Publications, Delhi.